

## Appendix 2.

# DYNAMIC PROBLEM FOR THE SYSTEM: A CAR – AN AIRBAG – AN OCCUPANT

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## 6. INTRODUCTION

This paper introduces a formulation of a dynamical problem of interaction between an occupant and an airbag.

Physical phenomena are under study that must be accounted for in this problem: adiabatic compression of gas that occurs when the airbag is being flattened, and the gas outflow from its holes. Formulae to describe these phenomena are given.

Systems of differential equations to solve the dynamic problem are also derived here.

Dependencies between the elastic response and the airbag volume, on one hand, and its degree of flattening, on the other hand, were calculated and are given in this paper. These dependencies were obtained using a numerical technique.

Results of solving a series of problems of interaction between the occupant and the airbag during a collision between a car and a barrier at various collision speeds are given here.

Possibilities of “soft” and “rigid” cars, as well as, variants of airbags with diameter of 750 mm and 625 mm were investigated. Research was carried out for the range of impact velocities from 8 m/sec up to 16 m/sec at distances between the occupant’s body and the airbag’s bottom from 200 mm up to 600 mm.

## 7. FORMULATION OF THE PROBLEM AND DERIVATION OF DIFFERENTIAL EQUATIONS

### 7.1. Formulation of the problem

Physical phenomena of interest are as follows.

A car and an occupant within the car are moving together at a constant speed. The occupant is located at the distance  $D_0$  from the bottom of the airbag. This is an initial position of the occupant.

At a moment of time  $t=0$ , the car hits a barrier, a sensor fires and activates a gas generator of the airbag. In time  $\tau$  the airbag is filled with a gas of known properties. This is an initial state of the airbag. As the airbag has holes in its surface, the gas starts flowing out of it. Thus, at the moment  $t=\tau$  the process of the airbag’s inflation is finished, pressure of the gas within the airbag is  $p_0$ , its temperature is  $T_0$ , and the gas is flowing out of the airbag through the holes that are in it.

The occupant, after the car hits the barrier, outruns the car moving by inertia and hits the airbag. The airbag flattens and creates a response force that affects the occupant. The motion of the occupant is slowed down by this force. The occupant’s body is deformed also, and its center of mass moves towards the airbag. In some time, the velocities of the occupant and the car become equal, but the airbag continues affecting the occupant and giving his body a speed of the opposite sign. This speed increases (by its absolute value) all the time while the occupant remains in contact with the airbag.

It may happen that the occupant will prevent the airbag from complete inflation. In this case the initial shape of the airbag will be a shape constrained by the occupant.

A schematic view of the system of interest is given on Fig. 7.1.

The dynamic problem of interaction between the occupant and the airbag can be thus formulated as follows.

Given:

- deceleration of the car as a function of time  $a(t)$ ;
- molar mass of the gas  $M$ ,
- adiabatic exponent  $\gamma$ ;
- pressure  $p_0$  and temperature  $T_0$  of the gas in its initial state;
- $p_a$ , atmospheric pressure;
- time  $\tau$  of the airbag’s filling with the gas;
- $s$ , area of holes in the airbag to let the gas out;
- $\mu$ , an empiric coefficient usually assumed to be 0.6;
- dependence between the airbag volume and its degree of flattening  $V(D)$ ;
- effective area of the airbag  $A(D)$  that transfers the gas pressure to the occupant, as a function of its degree of flattening;
- initial position of the occupant  $D_0$ ;
- effective mass of the occupant  $B$ ;

- stiffness of a conventional spring  $c$  that simulates elastic properties of the occupant's body.

On the basis of studies described in paper Investigation of axisymmetric film airbags, Appendix 1.

[ 9 ] I stated that geometric properties of the airbag filled with gas were little dependent on the gas pressure within. Therefore the problem's formulation assumes that the volume of the bag and its effective area depend only on its degree of flattening and do not depend on its internal pressure.

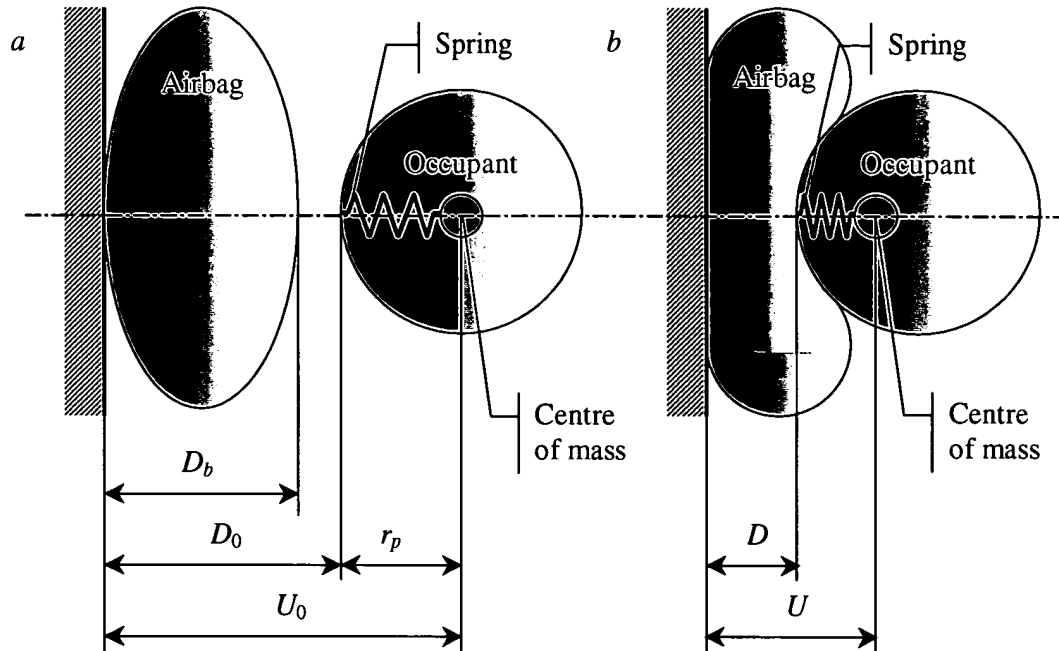


Fig. 7.1. A schematic view of the system to be analyzed. (a) initial position; (b) working position.

To be determined, as functions of time  $t$ :

- position of the center of mass of the occupant  $l(t)=U(t) - r_p$ ;
- velocity of the occupant's center of mass with respect to the car  $v(t)$ ;
- degree of flattening of the airbag  $D(t)$ ;
- mass of the gas located within the airbag  $m(t)$ ;
- pressure of the gas within the airbag  $p(t)$ ;
- temperature of the gas within the airbag  $T(t)$ ;
- coefficient of overload experienced by the occupant  $u(t)$ .

The solution is also to determine whether the fact of the occupant's collision with stiff parts of the car takes place, and if so, the velocity of this collision is to be found.

The problem is formulated and solved as unidimensional.

## 7.2. Gas state equation within the airbag

After the sensor fires, the gas generator fills the airbag with gas. Type of gas, its pressure and temperature are defined by the design of the gas generator.

The airbag has holes from which gas flows out.

After that, the occupant impacts the airbag, the airbag flattens, its volume decreases, and pressure of gas increases for some time. Then it falls again.

The process goes on very quickly, the system does not have enough time to absorb or to yield heat. Therefore we will consider the process to be adiabatic.

Tasks of this section are as follows:

- determine the initial mass of gas provided by the gas generator by the gas type, pressure and temperature, and by the initial volume of airbag;
- determine the gas pressure and temperature by the mass of gas within the airbag and the volume of the bag.

On the basis of Clapeyron – Mendeleev equation Investigation of axisymmetric film airbags, Appendix 1.

[ 10 ] that describes the state of perfect gas, the density of gas in its initial state can be expressed via its molar mass, initial pressure and initial temperature by the formula:

$$( 7.1 ) \quad \rho_0 = \frac{Mp_0}{RT_0};$$

where  $\rho_0$  is the density of gas in the initial state;

$M$  is the molar mass (for nitrogen it is  $M=0.0280134$  kg/mole);

$p_0$  is the initial pressure;

$T_0$  is the initial absolute temperature;

$R=8.31441$  J/(K·mole) is the universal gas constant.

Initial mass of gas within the airbag:

$$( 7.2 ) \quad m_0 = \rho_0 \cdot V_0;$$

where  $V_0$  is the initial airbag volume.

Let's consider the second problem of determining the gas pressure and temperature in the course of its adiabatic compression.

The Poisson equation Investigation of axisymmetric film airbags, Appendix 1.

[ 10 ] that describes this process yields the formulae:

$$( 7.3 ) \quad p = p_0 \varepsilon^\gamma;$$

$$( 7.4 ) \quad T = T_0 \varepsilon^{\gamma-1};$$

where  $p$  is pressure of gas in the current state;

$T$  is temperature of gas in the current state;

$\varepsilon$  is geometric ratio of compression;

$\gamma$  is adiabatic power.

Geometric ratio of compression is equal to ratio of the initial volume occupied by a certain amount of gas to the current volume that contains the same amount of gas. Geometric ratio of compression can be expressed via gas density by the following formula:

$$( 7.5 ) \quad \varepsilon = \frac{\rho}{\rho_0};$$

where  $\rho$  is density of gas in the current state.

In its turn, gas density in the current state can be determined using the following formula:

$$( 7.6 ) \quad \rho = \frac{m}{V}.$$

where  $V$  is volume of the airbag in the current state.

The adiabatic power can be determined using the number of degrees of freedom of gas molecules:

$$(7.7) \quad \gamma = \frac{i+2}{i};$$

where  $i$  is the number of degrees of freedom of the molecule.

For two-atom molecules (oxygen, nitrogen)  $i=5$ .

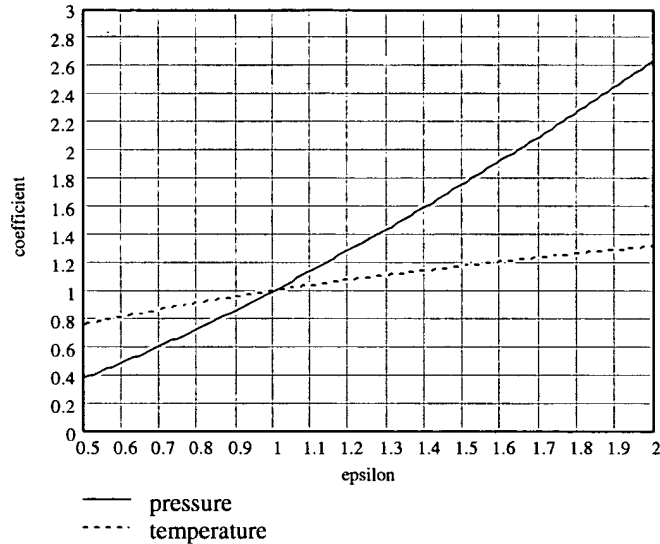


Fig. 7.2. Relative variation of pressure and absolute temperature of nitrogen vs. geometric ratio of compression  $\epsilon$  in an adiabatic process.

Substituting expressions of (7.5) and (7.6) to formulae (7.3) and (7.4), we will obtain:

$$(7.8) \quad p = p_0 \left( \frac{m}{\rho_0 V} \right)^\gamma;$$

$$(7.9) \quad T = T_0 \left( \frac{m}{\rho_0 V} \right)^{\gamma-1}.$$

### 7.3. Gas outflow from holes

The airbag has holes through which gas that fills it can flow out. Those are needed to prevent the occupant from being captured by the airbags any longer than it is necessary for their operation.

The gas outflow starts immediately after the airbag is filled, and it goes on as long as the pressure within the airbag exceeds that within the car.

The flow rate of a substance out of a hole can be described by the following formula:

$$(7.10) \quad \frac{dm}{dt} = v_g \rho s;$$

where  $v_g$  is speed of the substance's outflow at the outlet;

$\rho$  is density of the substance at the outlet;

$s$  is the cross-section area of the hole.

A formula is known for speed of gas outflow from a hole [11]:

$$(7.11) v_s = \sqrt{\frac{2p}{\rho} \frac{\gamma}{\gamma-1} \left(1 - \beta^{\frac{\gamma-1}{\gamma}}\right)};$$

where  $p$  is absolute gas pressure within the airbag;

$\gamma$  is adiabatic power;

$$(7.12) \beta = \frac{p_a}{p}.$$

At the outlet, gas that flows out has pressure  $p_a$ . At this pressure, density of gas can be expressed by the following formula:

$$(7.13) \rho_a = \rho \beta^{\frac{1}{\gamma}}.$$

Making use of formula ( 7.10 ), we will obtain a dependence to determine the flow rate of gas that flows out of the hole.

$$(7.14) \frac{dm}{dt} = s \beta^{\frac{1}{\gamma}} \sqrt{2\rho p \frac{\gamma}{\gamma-1} \left(1 - \beta^{\frac{\gamma-1}{\gamma}}\right)}.$$

Formula ( 7.14 ) can be represented as:

$$(7.15) \frac{dm}{dt} = \mu s k \sqrt{2\rho p};$$

where  $k$  is a coefficient that depends on the type of gas and the ratio between internal and external pressures. The formula for this coefficient is:

$$(7.16) k = \sqrt{\frac{\gamma}{\gamma-1} \left( \beta^{\frac{2}{\gamma}} - \beta^{\frac{\gamma+1}{\gamma}} \right)}.$$

Formula ( 7.15 ) contains also an empirical “orifice flow rate factor”  $\mu$  that depends on the compression rate of the stream and aerodynamic resistance. This one is assumed to be approximately 0.6.

Additional studies [ 12 ] show that formula ( 7.16 ) holds true only in the sonic interval of gas flow speed at

$$(7.17) \beta \geq \left( \frac{2}{\gamma+1} \right)^{\frac{\gamma}{\gamma-1}}$$

For a supersonic velocity of the gas flow, coefficient  $k$  keeps the same value as that achieved at

$$\beta = \left( \frac{2}{\gamma+1} \right)^{\frac{\gamma}{\gamma-1}}.$$

The diagram of dependence of coefficient  $k$  on  $\beta$  at  $\gamma=1.4$  (nitrogen) is shown on Fig. 7.3.

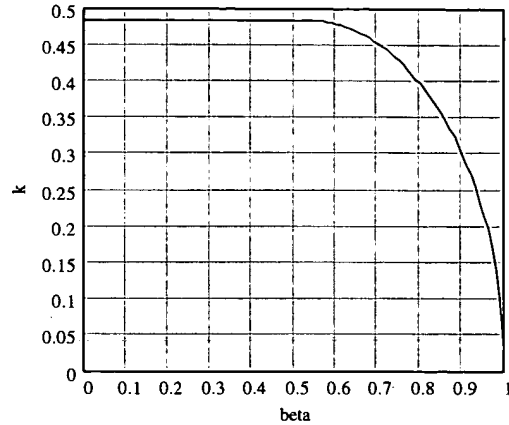


Fig. 7.3. Dependence  $k(\beta)$  for nitrogen

The dependence of nitrogen's flow rate to atmosphere on the internal airbag pressure is shown on Fig. 7.4. Source data for gas within the airbag are:  $p_0=114$  kPa,  $T_0=373.15$  K, adiabatic power  $\gamma=1.4$ . Atmospheric pressure is assumed to be  $p_a= 101.325$  kPa. The analysis accounts for 2 round holes of 2 cm diameter each. The orifice flow rate factor is  $\mu=0.6$ . The calculations are done using the formula ( 7.15 ).

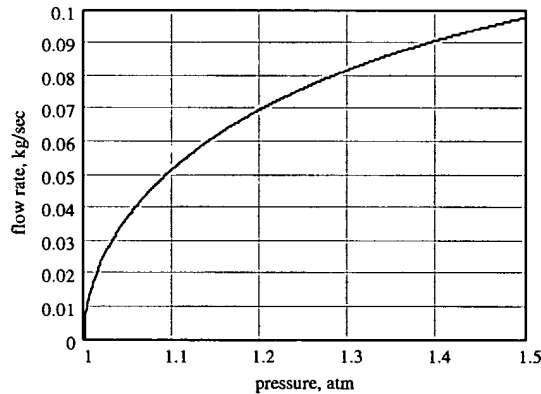


Fig. 7.4. Nitrogen flow rate to atmosphere from two round holes of 2 cm diameter vs. internal pressure within the airbag

Note that at normal conditions ( $p_a= 101.325$  kPa,  $T_0=293.15$  K) density of nitrogen is  $\rho=1.165$  kgf/m<sup>3</sup>.

#### 7.4. Derivation of differential equations

The problem that is posed studies together the following phenomena:

- dynamics of the occupant's body supported by the airbag;
- a process of adiabatic compression of the gas by inertial forces;
- a process of the gas outflow from the airbag.

Principal unknowns of the system of differential equations are as follows:

- $l$  is a coordinate that describes the position of the occupant's center of mass;
- $v$  is the velocity of the center of mass of the occupant with respect to the airbag's bottom (it is positive when the occupant is moving towards the bottom);
- $m$  is the mass of the gas in the airbag.

The coordinate that describes the position of the center of mass of the occupant is a difference between two values:

$$(7.18) \quad l = U - r_p;$$

where  $U$  is the distance between the center of mass of the occupant and the bottom of the airbag;  
 $r_p$  is the distance between the center of mass of the occupant and his exterior surface in the non-deformed state (see Fig. 7.1).

The coordinate is chosen in this way to exclude the distance  $r_p$  from consideration because it does not affect the solution.

The system of differential equations, itself pretty obvious, is as follows:

$$(7.19) \quad \frac{dl}{dt} = -v;$$

$$(7.20) \quad \frac{dv}{dt} = -a - \frac{X(l, m)}{B};$$

$$(7.21) \quad \frac{dm}{dt} = \frac{p_0 - p(l, m)}{p_0 - p_a} m_1 - \mu s k(l, m) \sqrt{2\rho(l, m)p(l, m)}$$

The following notations are used in addition to those already described:

$X(l, m)$  is the force of interaction between the occupant and the car (always positive);

$p(l, m)$  is the pressure within the airbag at the current moment of time;

$\rho(l, m)$  is the density of gas in the airbag at the current moment of time;

$k(l, m)$  is a factor.

Formulae that define these values are given below.

The principal problem is how to determine the degree of flattening of the airbag  $D$  depending on the coordinate of the center of mass of the occupant  $l$  and the amount of gas within the airbag  $m$ .

If the occupant and the airbag are in an equilibrium contact, then the response force of the airbag and the force in the conventional spring that supports the effective mass of the occupant must be equal to each other:

$$(7.22) \quad (D - l)c = A(D)(p - p_a).$$

Expressing the airbag pressure  $p$  by the formula (3.8) of Investigation of axisymmetric film airbags, Appendix 1.

[ 10 ], we obtain an equation:

$$(7.23) \quad (D - l)c = A(D) \left[ p_0 \left( \frac{m}{V(D)p_0} \right)^\gamma - p_a \right].$$

In this and subsequent equations the following holds:

$$(7.24) \quad \rho_0 = \frac{Mp_0}{RT_0}.$$

Resolving the equation ( 7.23 ) with respect to  $D$  at different combinations of  $l, m$ , we will obtain the desired dependence  $D_A(l, m)$ .

Let's consider the region in the space of variables wherein this dependence holds (Fig. 7.5). It is obvious that  $0 \leq D \leq D_b$ . As it follows from the equation ( 7.23 ), the minimum value of  $D=0$  corresponds to the boundary of the region described by the following dependence:

$$(7.25) \quad l = -\frac{A(0)}{c} \left[ p_0 \left( \frac{m}{V(0)p_0} \right)^\gamma - p_a \right].$$



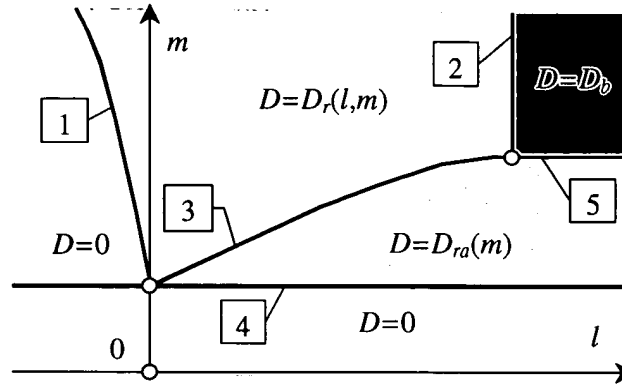


Fig. 7.5. A division of the plane  $l, m$  into areas of effectiveness of different formulae that determine the degree of flattening of the airbag  $D$

It is denoted as 1 on Fig. 7.5. At the maximum value of  $D=D_b$   $A(D_b)=0$ . Therefore the second boundary of the region can be found using a dependence:

$$(7.26) \quad l=D_b.$$

It is denoted as 2 on Fig. 7.5. Due to the airbag's softness, its excess pressure is not negative. Therefore there is a third boundary of the region defined by a dependence:

$$(7.27) \quad m = \rho_a V(l).$$

where  $\rho_a$  is the density of gas that fills the airbag at the atmospheric pressure. It is denoted as 3 on Fig. 7.5. One can make sure it's true by noticing that if there is no excess pressure, then  $l=D$  and

$$(7.28) \quad \rho_a = \rho_0 \left( \frac{p_a}{p_0} \right)^{\frac{1}{\gamma}}.$$

The area that has the boundaries 2 and 5 corresponds to the situation when the occupant does not touch the completely inflated airbag. This gives us

$$(7.29) \quad D=D_b.$$

The area with the boundaries 1 and 4 corresponds to the situation when the airbag is absolutely flat and the occupant touches its stiff bottom. This gives us

$$(7.30) \quad D=0.$$

For these three areas the pressure within the airbag exceeds the atmospheric pressure, and due to this fact the airbag has a certain shape. Other areas correspond to the airbag that is not completely inflated and, strictly speaking, does not have any particular shape. Though, for these areas we can also derive credible formulae to determine  $D$ .

On Fig. 7.5,  $D_{ra}(m)$  denotes a root of the equation

$$(7.31) \quad m = \rho_a V(D).$$

The formulae (7.32) to (7.35) represent all the logic described above.

$$(7.32) \quad D_1(l, m) = \begin{cases} 0 & \text{for } 0 \leq m < V(0) \rho_a \left( 1 - \frac{cl}{p_a A(0)} \right)^{\frac{1}{\gamma}}; \\ D_r(l, m) & \text{for } V(0) \rho_a \left( 1 - \frac{cl}{p_a A(0)} \right)^{\frac{1}{\gamma}} \leq m \end{cases}$$

$$(7.33) \quad D_2(l, m) = \begin{cases} 0 & \text{for } 0 \leq m < V(0)\rho_a \\ D_m(m) & \text{for } V(0)\rho_a \leq m < V(l)\rho_a; \\ D_r(l, m) & \text{for } V(l)\rho_a < m \end{cases}$$

$$(7.34) \quad D_3(m) = \begin{cases} 0 & \text{for } 0 \leq m < V(0)\rho_a \\ D_m(m) & \text{for } V(0)\rho_a \leq m < V(D_b)\rho_a \\ D_b & \text{for } V(D_b)\rho_a < m \end{cases}$$

$$(7.35) \quad D(l, m) = \begin{cases} D_1(l, m) & \text{for } l \leq 0 \\ D_2(l, m) & \text{for } 0 \leq l < D_b. \\ D_3(m) & \text{for } D_b \leq l \end{cases}$$

Similarly, the problem of determining the pressure within the airbag can be solved. This pressure is either equal to the atmospheric pressure or can be expressed by the formulae (7.36) – (7.38).

$$(7.36) \quad P_0(m) = p_0 \left( \frac{m}{V(0)\rho_0} \right)^\gamma;$$

$$(7.37) \quad P_D(l, m) = p_0 \left( \frac{m}{V(D_r(l, m))\rho_0} \right)^\gamma;$$

$$(7.38) \quad P_B(m) = p_0 \left( \frac{m}{V(D_b)\rho_0} \right)^\gamma.$$

Areas of effectiveness of the formulae are shown on Fig. 7.6.

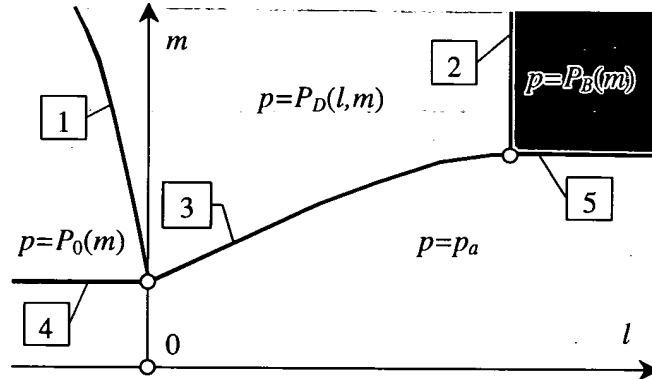


Fig. 7.6. A division of the plane  $l, m$  into areas of effectiveness of different formulae to determine the pressure in the airbag  $p$

The formulae (7.39) to (7.42) represent the logic shown visually on Fig. 7.6.

$$(7.39) \quad p_1(l, m) = \begin{cases} p_a & \text{for } 0 \leq m < V(0)\rho_a \\ P_0(m) & \text{for } V(0)\rho_a \leq m < V(0)\rho_a \left( 1 - \frac{cl}{p_a A(0)} \right)^{\frac{1}{\gamma}}; \\ P_D(l, m) & \text{for } V(0)\rho_a \left( 1 - \frac{cl}{p_a A(0)} \right) \leq m \end{cases}$$

$$(7.40) \quad p_2(l, m) = \begin{cases} p_a & \text{for } 0 \leq m < V(l)\rho_a; \\ P_D(l, m) & \text{for } V(l)\rho_a \leq m \end{cases};$$

$$(7.41) \quad p_3(m) = \begin{cases} p_a & \text{for } 0 \leq m < V(D_b)\rho_a; \\ P_B(m) & \text{for } V(D_b)\rho_a \leq m \end{cases};$$

$$(7.42) \quad p(l, m) = \begin{cases} p_1(l, m) & \text{for } l \leq 0 \\ p_2(l, m) & \text{for } 0 \leq l < D_b; \\ p_3(m) & \text{for } D_b \leq l \end{cases}.$$

The force with which the airbag affects the occupant can be expressed by a formula:

$$(7.43) \quad X_D(l, m) = A(D_r(l, m)) \left( p_0 \left( \frac{m}{V(D_r(l, m))} \right)^\gamma - p_a \right);$$

It is proportional to the coordinate  $l$  when the airbag is flat, and it is zero when there is no contact between the occupant and the airbag.

A division of the plane  $l, m$  into areas of effectiveness of different formulae for the force of interaction between the occupant and the airbag is shown on Fig. 7.7.

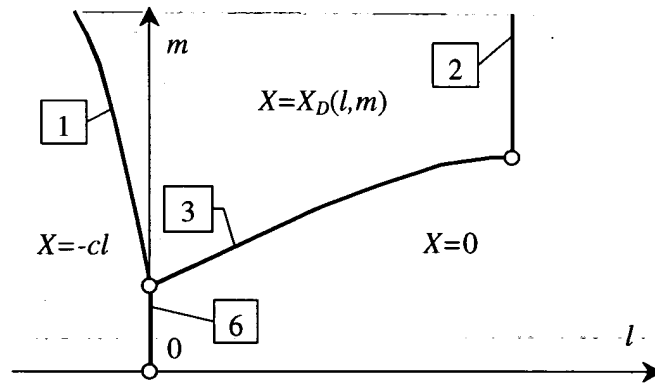


Fig. 7.7. A division of the plane  $l, m$  into areas of effectiveness of different formulae for the force of interaction between the occupant and the airbag  $X$

The same logic can be described analytically by the following formulae:

$$(7.44) \quad X_1(l, m) = \begin{cases} -cl & \text{for } 0 \leq m < V(0)\rho_a \left( 1 - \frac{cl}{p_a A(0)} \right)^{\frac{1}{\gamma}}; \\ X_D(l, m) & \text{for } V(0)\rho_a \left( 1 - \frac{cl}{p_a A(0)} \right)^{\frac{1}{\gamma}} \leq m \end{cases};$$

$$(7.45) \quad X_2(l, m) = \begin{cases} 0 & \text{for } 0 \leq m < V(l)\rho_a; \\ X_D(l, m) & \text{for } V(l)\rho_a \leq m \end{cases};$$

$$(7.46) \quad X(l, m) = \begin{cases} X_1(l, m) & \text{for } l \leq 0 \\ X_2(l, m) & \text{for } 0 \leq l < D_b; \\ 0 & \text{for } D_b \leq l \end{cases};$$

The equation (7.21) determines the speed of change of the gas mass within the airbag. The first term of the right part of this equation describes the inflow of the gas from the gas generator. It is

a simplified dependency where  $m_1$  is the speed of increase of the gas mass within the airbag if the pressure within is not equal to the atmospheric pressure. This speed is a function of time and is chosen according to the requirement that the amount of gas needed to fill the airbag come to it for the time  $\tau$ :

$$(7.47) \quad m_1 = \begin{cases} \frac{\rho_0 V(D_i)}{\tau} & \text{for } 0 \leq t \leq \tau \\ 0 & \text{for } \tau < t \end{cases}$$

The coefficient  $\frac{p_0 - p(l, m)}{p_0 - p_a}$  prevents the gas from coming into the airbag if the pressure in it achieves the value of  $p_0$ . The second term describes the gas outflow from the airbag through holes. This problem is referred to in detail in Investigation of axisymmetric film airbags, Appendix 1.

[ 10 ]. Here only most necessary formulae are given:

$$(7.48) \quad k(l, m) = \sqrt{\frac{\gamma}{\gamma-1} \left( \beta(l, m)^{\frac{2}{\gamma}} - \beta(l, m)^{\frac{\gamma+1}{\gamma}} \right)};$$

$$(7.49) \quad \beta(l, m) = \begin{cases} \frac{p_a}{p(l, m)} & \text{for } \left( \frac{2}{\gamma+1} \right)^{\frac{\gamma}{\gamma-1}} \leq \frac{p_a}{p(l, m)} \leq 1 \\ \left( \frac{2}{\gamma+1} \right)^{\frac{\gamma}{\gamma-1}} & \text{for } 0 \leq \frac{p_a}{p(l, m)} < \left( \frac{2}{\gamma+1} \right)^{\frac{\gamma}{\gamma-1}} \end{cases};$$

$$(7.50) \quad \rho(l, m) = \frac{m}{V(D(l, m))}.$$

The initial conditions for the solution of the system of differential equations are as follows:

$$(7.51) \quad l = D_0; v = 0; m = 0.$$

The temperature of the gas within the airbag can be determined, after the problem is solved, by the formula:

$$(7.52) \quad T = T_0 \left( \frac{\rho(l, m)}{\rho_0} \right)^{\gamma-1}.$$

The overload coefficient for the occupant is determined by the formula:

$$(7.53) \quad u = \frac{X(l, m)}{gB};$$

where  $g$  is the free fall acceleration.

### 7.5. Dependencies $A(D)$ and $V(D)$

An airbag is under consideration with the following properties (provided by Rino Castelli):

- Radius of the flat airbag is 0.375 m
- Film thickness is 0.1 mm
- Elasticity modulus is 3600 MPa
- Poisson ratio is 0.39;
- Occupant's radius is 0.8 m;

- Excess pressure within the airbag is 14 kPa.

Calculations were done on the basis of the paper [ 8 ] using a numerical technique and the software MathCAD-2000 Pro.

It was found out that the maximum thickness of the airbag in its inflated state was  $D_b=0.3535$  m, and its volume  $V(D_b)=0.06566$  m<sup>3</sup>. A series of 11 calculations was performed, and it yielded values of the effective area  $A(D)$  and the volume of the airbag  $V(D)$  at different flattening degree coefficients  $D$ . Using these values and the method of least squares, approximation polynomial functions were obtained:

$$(7.54) \quad A(D)=1.524864D^3+0.04385D^2-1.12712D+0.3256;$$

$$(7.55) \quad V(D)=0.24658D^4+0.096566D^3-0.57555D^2+0.327141D+0.0138212.$$

Fig. 7.8 and Fig. 7.9 show comparisons of the numerical results with the approximation polynomials ( 7.54 ), ( 7.55 ).

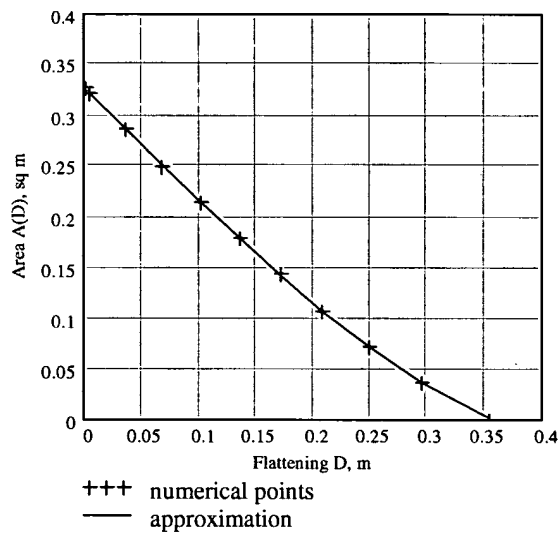


Fig. 7.8. A dependence of the effective area of the airbag  $A(D)$  on its flattening

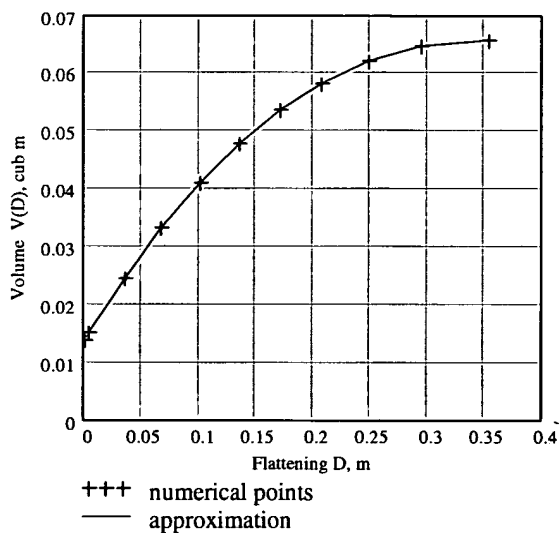


Fig. 7.9. A dependence of the volume of the airbag  $V(D)$  on its flattening

## 7.6. Solution of examples

Below, solutions are given for the problem of interaction between the occupant and the airbag during the collision of the car with a barrier.

Calculations were done using formulae of Section 7, a numerical technique, and the MathCAD-2000 Pro software.

The source data are as follows.

The initial velocity of the car and the occupant is 55 km/hr (40 km/hr, 30 km/hr).

The deceleration of the car during the collision with the barrier is 30g.

The initial position of the occupant is 0.32 m from the bottom of the airbag.

The mass of the occupant's top body 45 kilograms.

The "radius" of the occupant is 0.8 m.

The "stiffness" of the occupant's body is 400 kN/m.

Data of the safety airbag:

- Radius of the flat airbag is 0.375 m;
- Film thickness is 0.1 mm;
- Elasticity modulus is 3600 МПа;
- Poisson ratio is 0.39;
- Gas that fills the airbag is nitrogen at the temperature of 100°C;
- Excess pressure in the final phase of the airbag's filling is 14 kPa;
- Time of the airbag's filling with gas is 0.015 sec;
- Two outlet holes for gas 2 cm in diameter each.

Results of the calculations are given as diagrams below.

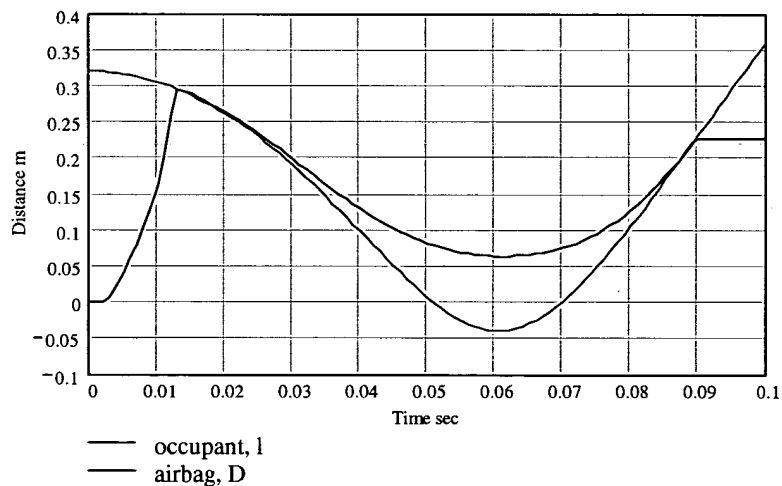


Fig. 7.10. Diagrams of motion of the occupant's center of mass  $l$  with respect to the car and variation of the airbag's thickness  $D$  at the speed of 55 km/hr.

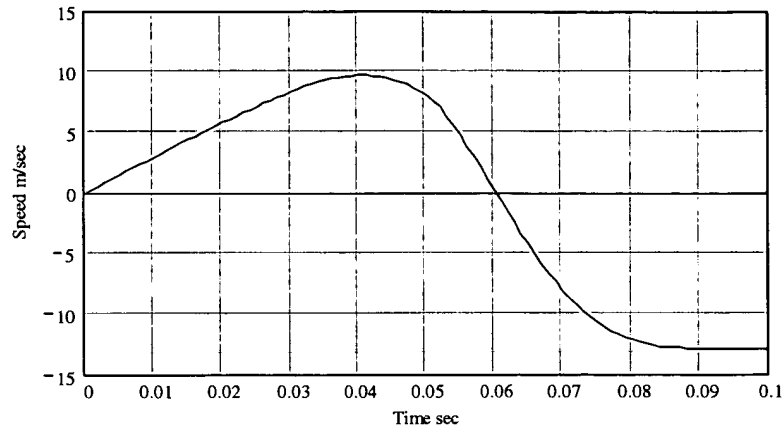


Fig. 7.11. A diagram of the velocity of the occupant's center of mass  $v$  with respect to the car at the speed of 55 km/hr.

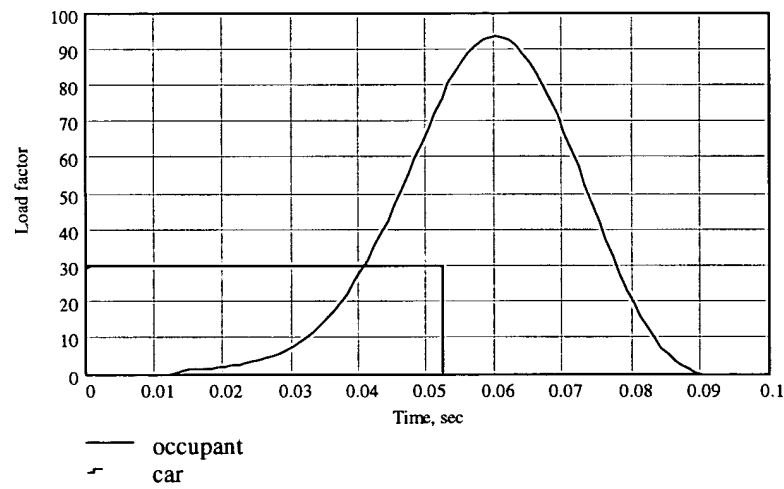


Fig. 7.12. Overload coefficients of the occupant and the car at the speed of 55 km/hr.

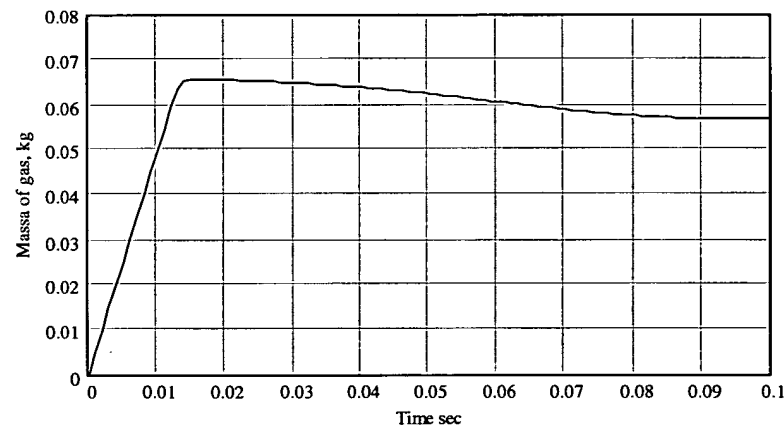


Fig. 7.13. A diagram of variation of the gas mass  $m$  in the airbag at the speed of 55 km/hr.

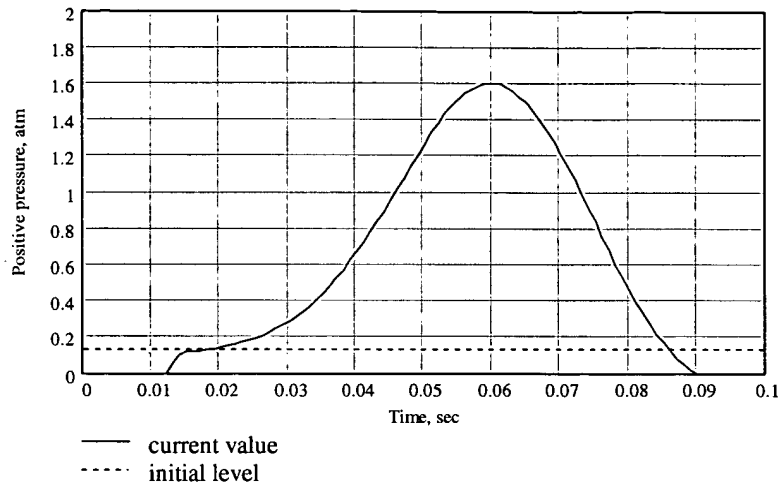


Fig. 7.14. A diagram of variation of the gas excess pressure in the airbag  $p-p_a$  at the speed of 55 km/hr.

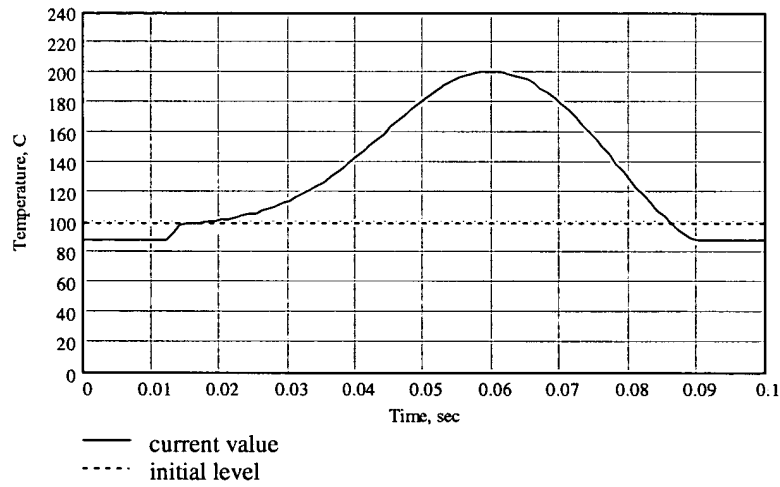


Fig. 7.15. A diagram of variation of the gas temperature in the airbag °C at the speed of 55 km/hr.



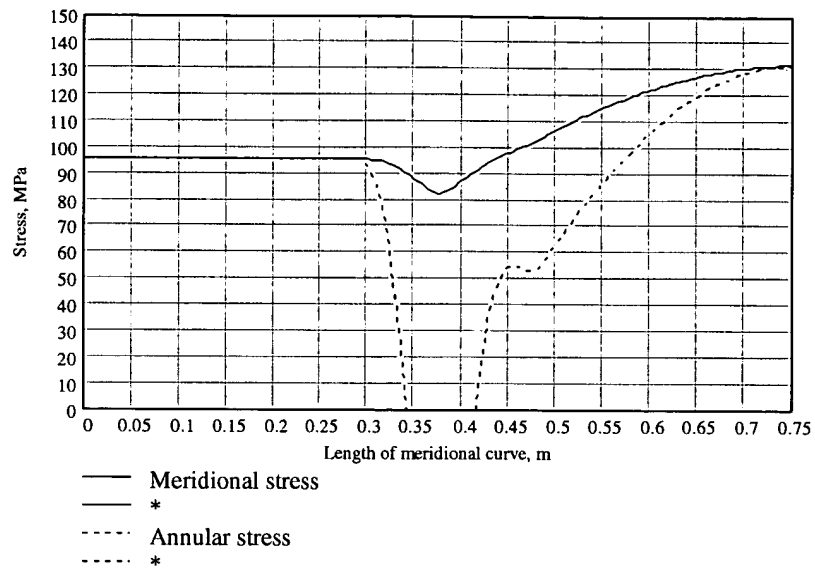


Fig. 7.16. Stress at the maximum pressure, at the speed of 55 km/hr

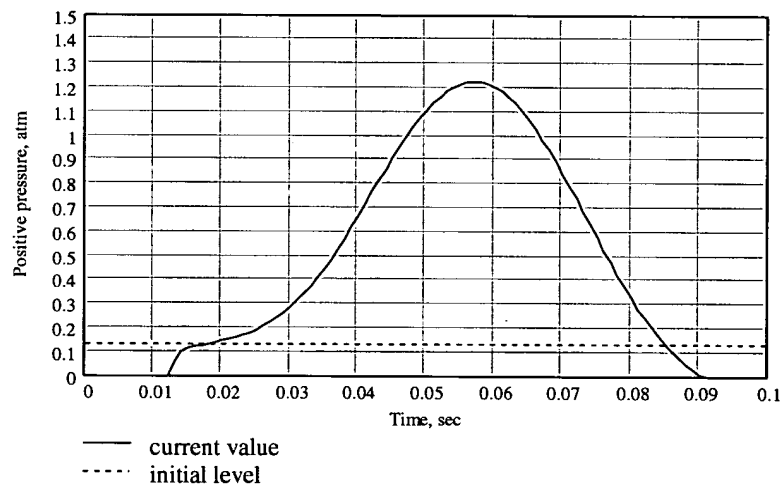


Fig. 7.17. A diagram of variation of the excess gas pressure in the airbag at the speed of 40 km/hr

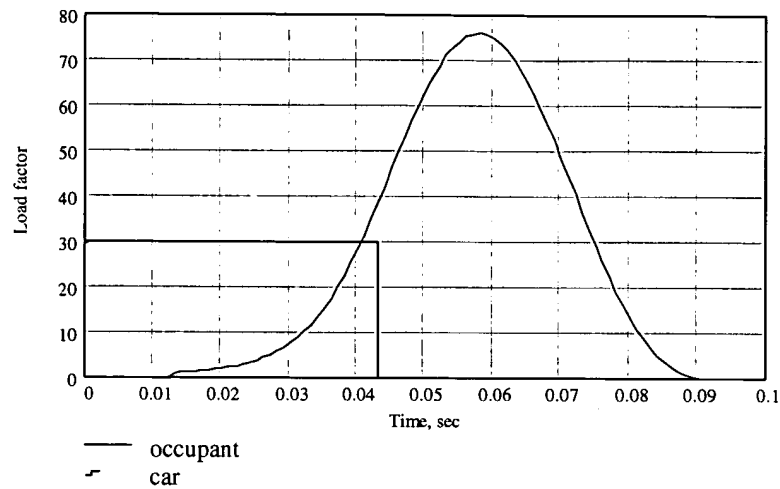


Fig. 7.18. Coefficients of overload of the occupant and the car at the speed of 40 km/hr

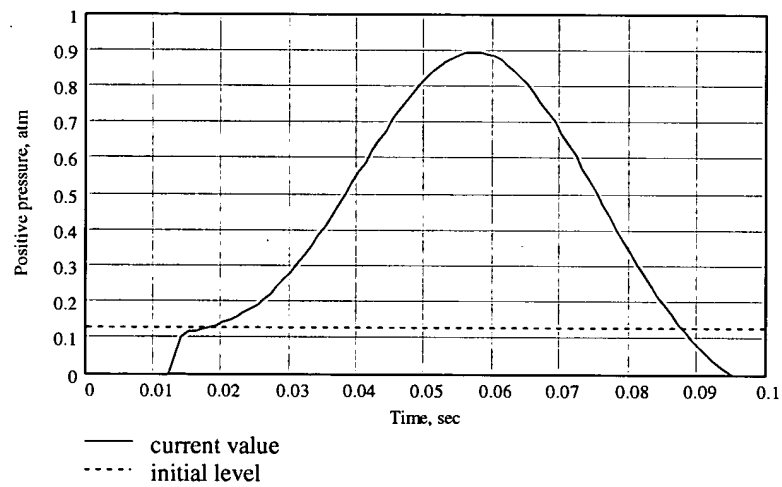


Fig. 7.19. A diagram of variation of the excess gas pressure at the speed of 30 km/hr.

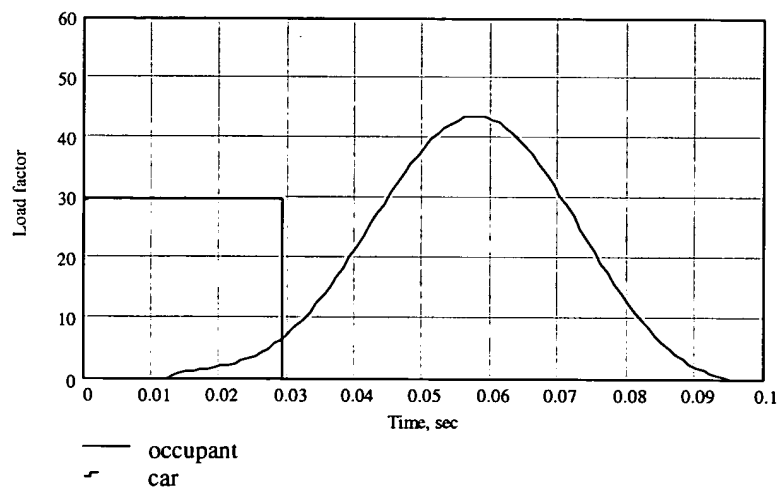


Fig. 7.20. Coefficients of overload of the occupant and the car at the speed of 30 km/hr

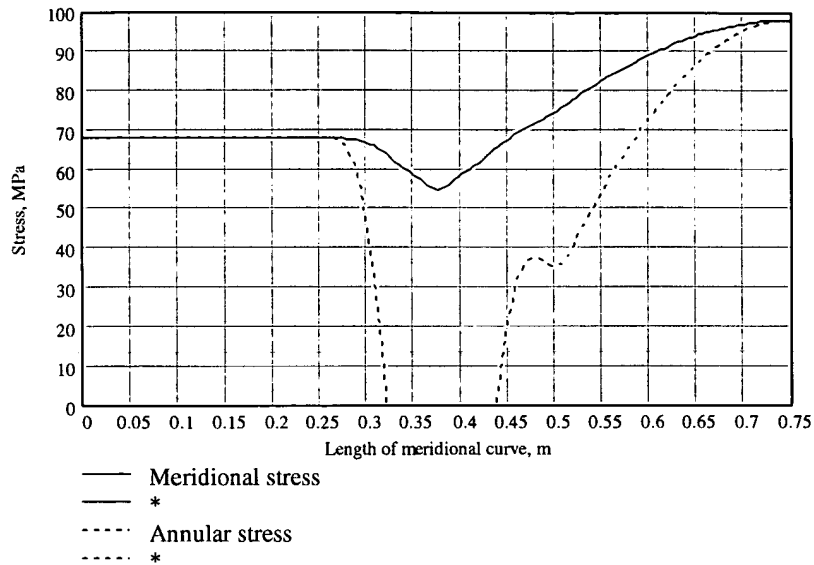


Fig. 7.21. Stress at the maximum pressure at the speed of 30 km/hr

## 1. PROPERTIES OF AIRBAGS

### 1.1. Comparison of deformed and stressed airbag's states at a constant volume and a constant amount of gas.

Comparison of dependences of reduced area  $A(D)$ , volume of the airbag  $V(D)$  and relative stresses  $s(D)$  at a constant pressure within the airbag (variant a), and a constant amount of gas (variant b) was executed for parameters of the airbag as follows:

- Radius of the completely flattened airbag is 0.375 m;
- Thickness of the film is 0.1 mm;
- Modulus of elasticity is 3600 MPa;
- Poisson's ratio is 0.39;
- "Occupant's radius " is 0.8 m.

According to the proceeding Investigation of axisymmetric film airbags, Appendix 1.

[ 9 ] , the maximal thickness of the airbag, being in inflated state, with the above mentioned parameters is  $D_b=0.3535$  m, ant its volume is  $V(D_b)=0.06566$  m<sup>3</sup>.

In case of the variant *a*, the dependences were calculated at the overpressure 14 Pa within the airbag. In case of the variant *b*, the dependences were calculated under condition that the airbag inflated with nitrogen at the amount of 0.06837 kg, and during the process of flattening the adiabatic process takes place. This amount of gas at the temperature 100 degrees of Celsius and at a atmospheric pressure  $p_a=101.325$  kPa creates an overpressure 14 kPa within the completely inflated airbag (with the volume of 0.06566 m<sup>3</sup>).

The calculations were based on the proceeding [ 8 ], applying calculus of approximation with the program Mathcad-2000 professional. Two series of calculations were carried out for effective area  $A(D)$  and the airbag's volume  $V(D)$  at different indexes of flattening  $D$ . Using these results and applying a technique of least squares, the following polynomial functions were obtained:

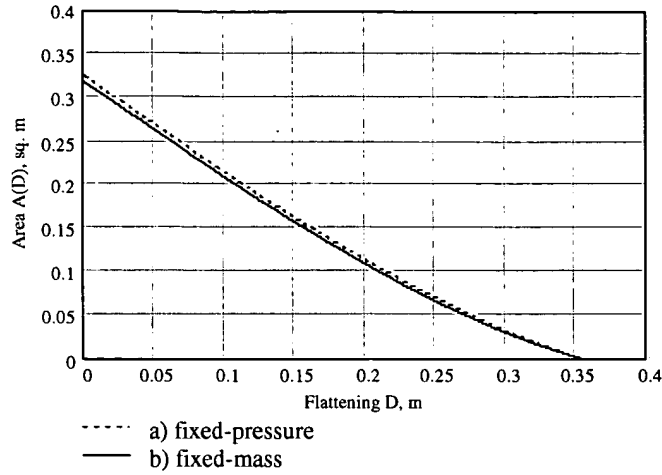


Fig. 1.1. The dependence of effective area  $A(D)$  on index of flattening  $D$  at the fixed pressure  $p=14$  kPa (without taking into account elastic flexibility of the material) and at the fixed mass of gas  $m=0.06837$  kg (taking into account elastic flexibility of the material).

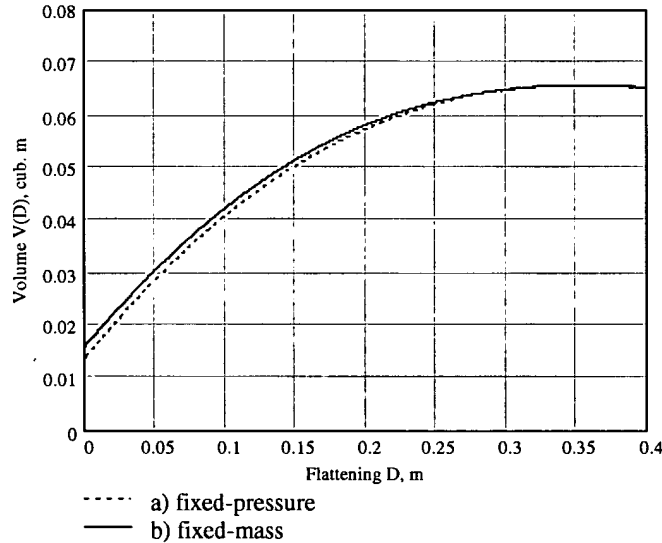


Fig. 1.2. The dependence of airbag's volume  $V$  on index of flattening  $D$  at the fixed pressure  $p=14$  kPa (without taking into account elastic flexibility of the material) and at the fixed mass of gas  $m=0.06837$  kg (taking into account elastic flexibility of the material).

for the variant  $a$ :

$$(1.1) \quad A(D)=1.524864D^3+0.04385D^2-1.12712D+0.3256;$$

$$(1.2) \quad V(D)=0.24658D^4+0.096566D^3-0.57555D^2+0.327141D+0.0138212;$$

for the variant  $b$ :

$$(1.3) \quad A_a(D)=1.806640D^3-0.05807D^2-1.10527D+0.3182;$$

$$(1.4) \quad V_a(D)=0.45166D^4-0.019357D^3-0.552634D^2+0.318162D+0.01605017.$$

On the Fig. 7.8 and Fig. 7.9 the juxtaposing of dependences for the variants  $a$  and  $b$  is shown.

It is seen that the curved lines for the variant  $a$  and  $b$  are similar. Though, in the future calculations the curves, obtained for the variant  $b$ , will be used, as more precise.

The stresses within the airbag's film have more complicated character.

Three characteristic zone of pressure can be singled out. They are as follows: the top centre of the airbag, the bottom centre of the airbag, and equator zone where there are the connective joint of circular work pieces of the airbag. For these zones the diagrams of relative meridional stresses  $s$  are shown on the Fig. 1.3.

Relative stresses are dimensionless quantities and they are expressed by the following formula:

$$(1.5) \quad s = \sigma/p;$$

where  $\sigma$  is stress;  $p$  is overpressure within the airbag.

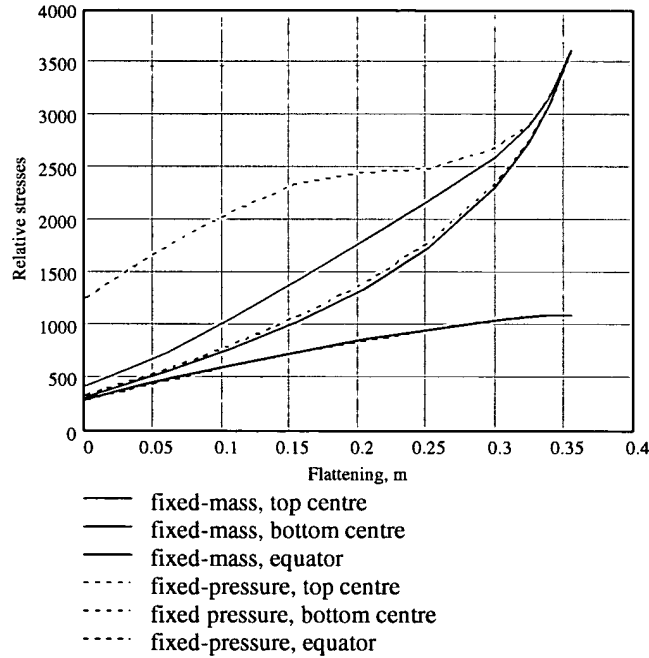


Fig. 1.3. Dependence of relative meridional stresses within the airbag's film on a degree of flattening  $D$  at the fixed pressure  $p=14$  kPa (without taking into account elastic flexibility of the material) and at the fixed mass of gas  $m=0.06837$  kg (taking into account elastic flexibility of the material).

The most stable stresses are in the equator zone. The relative values of these stresses are practically equal to each other at a fixed pressure and at a fixed mass of gas. The above stresses are proportionate to pressure.

Less stable stresses are in the bottom centre zone of the airbag. The relative values of such stresses are insignificantly different at a fixed pressure and at a fixed mass of gas. In the central zone of the airbag's bottom the meridional stresses are almost proportional to pressure. At that it is better to use the dependence for a constant mass due to better describing by this regime of the real conduction of the airbag at the moment of impact of the car with the barrier.

The relative values of stresses in the top centre zone of the airbag are significantly different at a fixed pressure and at a fixed mass of gas. It means that in this zone the meridional stresses have nonlinear dependence on pressure within the airbag. Using even the dependence for a constant mass, the linear formula (1.5) will calculate only approximate values, that should be additionally improved.

Sufficiently good results are obtained by the beneath mentioned approximate formulas for pressures similar to pressures in the adiabatic process.:

$$(1.6) \quad \sigma_0 = s_{01} p;$$

$$(1.7) \quad \sigma' = s'_{11} p;$$

$$(1.8) \quad \sigma = s_1 p + (s_1 - s_1) \sqrt{(2p_1 - p)p};$$

where  $\sigma_0$  is stress in the equator zone of the airbag, as a function of pressure;  
 $\sigma'$  is stress in the bottom centre zone of the airbag, as a function of pressure;  
 $\sigma$  is stress in the top centre zone of the airbag, as a function of pressure;  
 $p$  is overpressure within the airbag;  
 $p_1$  is overpressure within the airbag, conducted in adiabatic process;  
 $s_{01}$  is relative pressure in the equator zone of the airbag at a pressure  $p_1$ ;  
 $s_1$  is relative pressure in the top centre zone of the airbag at a pressure  $p_1$ ;  
 $s'_1$  is relative pressure in the bottom centre zone of the airbag at a pressure  $p_1$ .

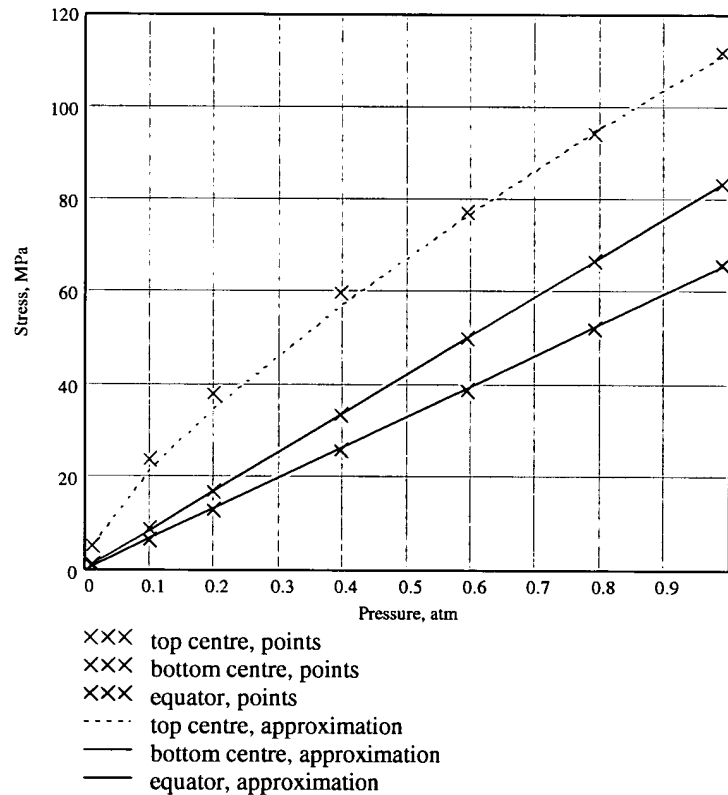


Fig. 1.4. Approximation of dependence of stresses within the airbag's material to inside pressure  
On the Fig. 1.4 the quality of approximate formulas( 1.6) –( 1.8) is graphically shown in case of the airbag's diameter of 750 mm with the "occupant's radius" of 800 mm. Let us consider a case, where a degree of flattening corresponds to the effective area 0.2 m<sup>2</sup>. Thus  $p_1 = 0.855$  atm,  $s_{01} = 654.2$ ;  $s_1 = 1156.2$ ;  $s'_1 = 833.7$ .

## 1.2. The dependences for airbag's diameter of 750 mm

The data, discussed in this section, are based on the results; obtained by two series of calculations for the airbag with the parameters as follows:

- Radius of the completely flattened airbag is 0.375 m;
- Thickness of the film is 0.1 mm;
- Modulus of elasticity is 3600 MPa;
- Poisson's ratio is 0.39.

One of the series of calculations was done for the “occupant’s radius”  $R=0.4$  m, the second one was done for the “occupant’s radius”  $R=0.8$  m.

At the overpressure 14 KPa within thickness of the airbag at the inflated state is  $D_b=0.3535$  m, and its volume is  $V(D_b)=0.06566$  m<sup>3</sup>.

The dependences were calculated at 0.06837kg of nitrogen in the airbag, and the adiabatic process takes place during flattening of the airbag. This amount of nitrogen creates overpressure of 14 kPa within the completely inflated airbag at the temperature 100 degrees of Celsius and at the atmospheric pressure  $p_a=101.325$  kPa.

The polynomial dependences of efficient area  $A$  of the airbag and its volume  $V$  to the degree of flattening  $D$  at a fixed mass of gas were obtained:

For  $R=0.4$  m:

$$(1.9) \quad A_{7540}(D)=1.001294 D^3 + 0.066357 D^2 - 0.958487 D + 0.285391;$$

$$(1.10) \quad V_{7540}(D)=0.44768 D^4 - 0.1709 D^3 + 0.38008 D^2 + 0.25368 D + 0.024038.$$

For  $R=0.8$  m:

$$(1.11) \quad A_{7580}(D)=1.806640 D^3 - 0.05807 D^2 - 1.10527 D + 0.3182;$$

$$(1.12) \quad V_{7580}(D)=0.45166 D^4 - 0.019357 D^3 - 0.552634 D^2 + 0.318162 D + 0.01605017.$$

These dependences are shown on the diagrams of Fig. 1.5 and Fig. 1.6.

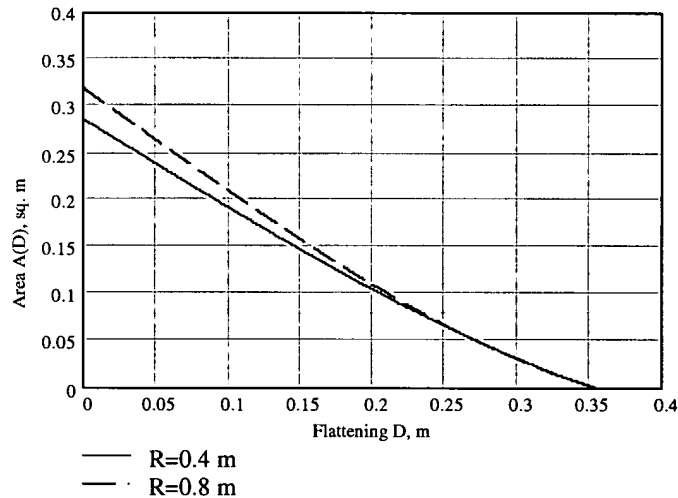


Fig. 1.5. The dependence of efficient area  $A$  of the airbag on a degree of its flattening  $D$  at two values of “occupant’s radius”  $R=0.4$  m and  $R=0.8$  m

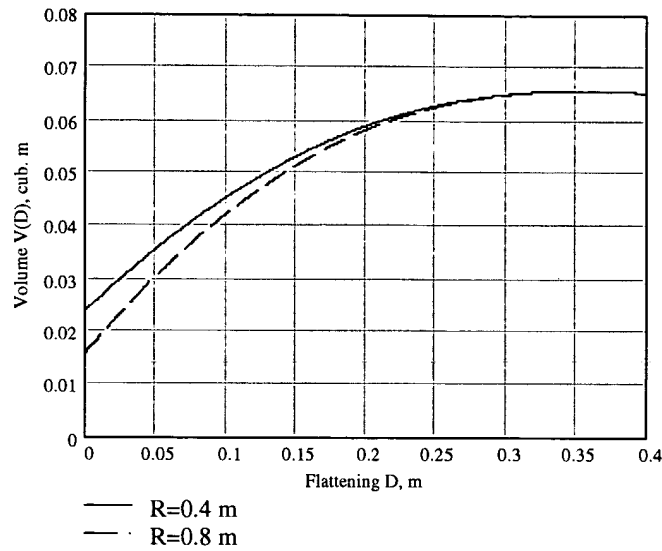


Fig. 1.6. The dependence of volume  $V$  of the airbag on a degree of its flattening  $D$  at two values of “occupant’s radius”  $R=0.4$  m and  $R=0.8$  m

Also the dependences of relative meridional stresses within the airbag’s film on a degree of its flattening  $D$  at the values of “occupant’s radius”  $R=0.4$  m and  $R=0.8$  m and at the fixed mass of gas  $m=0.06837$  kg were obtained. The diagrams of the dependences are shown on Fig. 1.7 and Fig. 1.8.

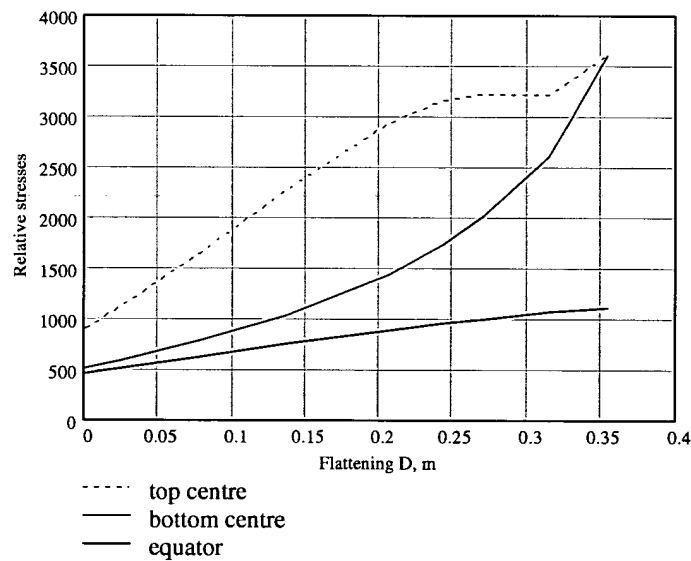


Fig. 1.7. The dependence of relative meridional stresses within the airbag’s film on a degree of its flattening  $D$  at the “occupant’s radius”  $R=0.4$  m



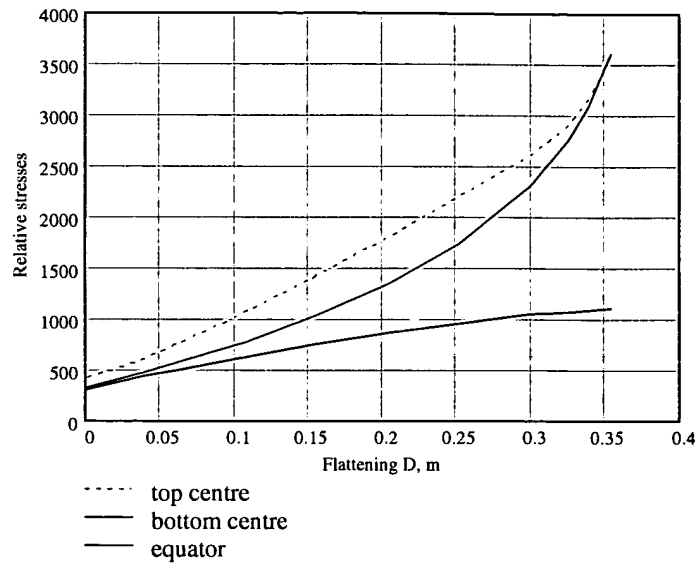


Fig. 1.8. The dependence of relative meridional stresses within the airbag's film on a degree of its flattening  $D$  at the "occupant's radius"  $R=0.8\text{m}$

### 1.3. Dependences for the airbag's diameter of 625mm

The same calculations were done for the airbag with the parameters as follows:

- Radius of the completely flattened airbag is 0.3125 m;
- Thickness of the film is 0.1 mm;
- Modulus of elasticity is 3600 MPa;
- Poisson's ratio is 0.39.

One of the series of calculations is done for the "occupant's radius"  $R=0.4\text{ m}$ , the second one is done for the "occupant's radius"  $R=0.8\text{ m}$ .

At the overpressure 14 kPa within thickness of the airbag at the inflated state is  $D_b=0.2935\text{ m}$ , and its volume is  $V(D_b)=0.037876\text{ m}^3$ .

The dependences were calculated at 0.03944 kg of nitrogen in the airbag, and the adiabatic process holds during flattening of the airbag. This amount of nitrogen creates overpressure of 14 kPa within the completely inflated airbag at the temperature 100 degrees of Celsius and at the atmospheric pressure  $p_a=101.325\text{ kPa}$ .

The polynomial dependences of efficient area  $A$  of the airbag and its volume  $V$  on a degree of flattening  $D$  at a fixed mass of gas were obtained:

For  $R=0.4\text{ m}$ :

$$(1.13) \quad A_{6240}=1.852226D^3-0.175723D^2-0.811144D+0.205879;$$

$$(1.14) \quad V_{6240}=0.52362D^4-0.12176D^3-0.35937D^2+0.18946D+0.01241.$$

For  $R=0.8\text{ m}$ :

$$(1.15) \quad A_{6280}=1.47676D^3+0.469628D^2-1.109566D+0.24737;$$

$$(1.16) \quad V_{6280}=0.66871D^4-0.08112D^3-0.47094D^2+0.229779D+0.0080927.$$

The diagrams of the dependences are shown on Fig. 1.9 and Fig. 1.10.

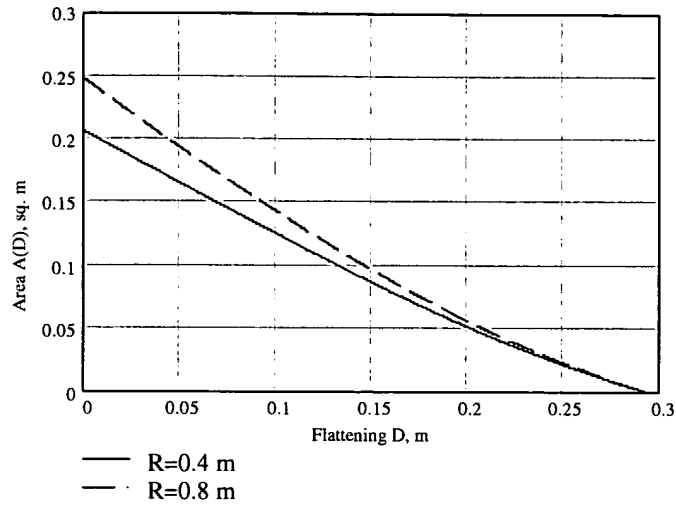


Fig. 1.9. The dependence of efficient area  $A$  of the airbag on a degree of its flattening  $D$  at two values of “occupant’s radius”  $R=0.4$  m and  $R=0.8$  m

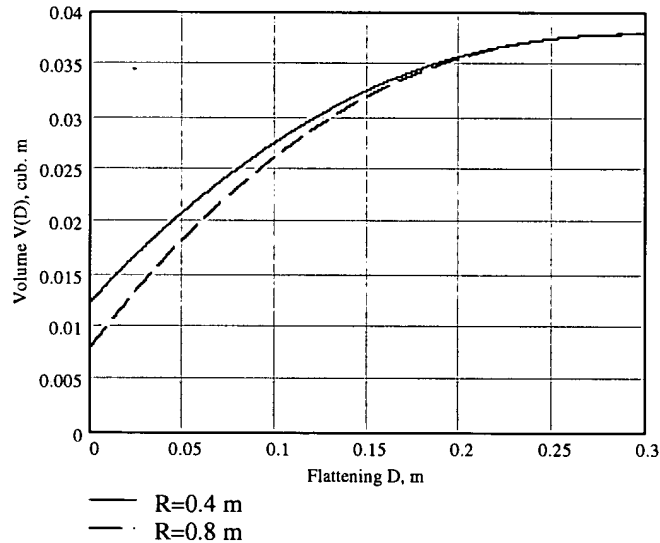


Fig. 1.10. The dependence of volume  $V$  of the airbag on a degree of its flattening  $D$  at two values of “occupant’s radius”  $R=0.4$  m and  $R=0.8$  m

Also the dependences of relative meridional stresses within the airbag’s film on a degree of its flattening  $D$  at the values of “occupant’s radius”  $R=0.4$  m and  $R=0.8$  m and at the fixed mass of gas  $m=0.0944$  kg were obtained. The diagrams of the dependences are shown on Fig. 1.11 and Fig. 1.12.

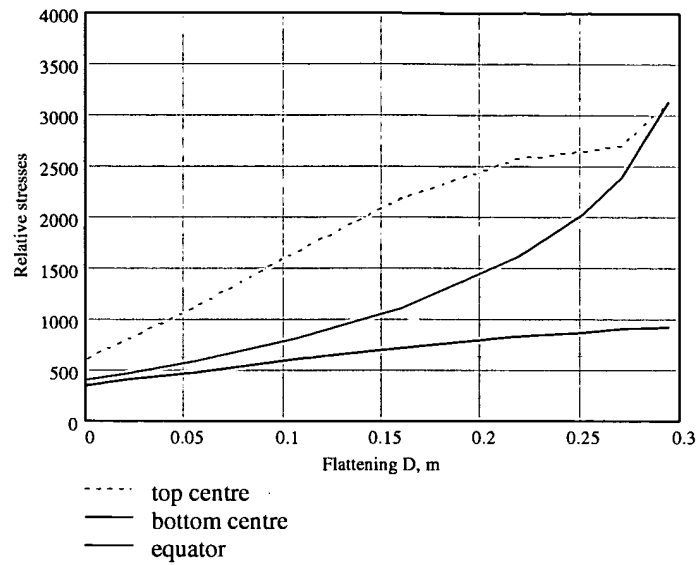


Fig. 1.11. The dependence of relative meridional stresses within the airbag's film on a degree of its flattening  $D$  at the "occupant's radius"  $R=0.4$  m

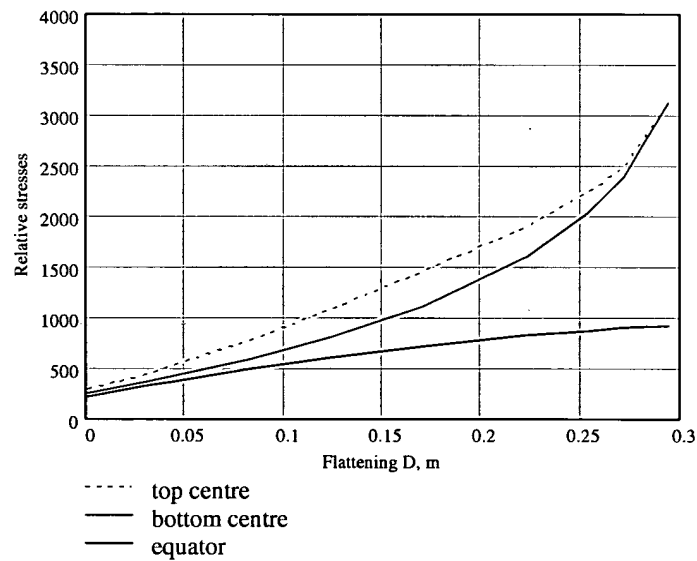


Fig. 1.12. The dependence of relative meridional stresses within the airbag's film on a degree of its flattening  $D$  at the "occupant's radius"  $R=0.8$  m

## 8. DYNAMIC OF PROCESSES

### 8.1. The base accelerogram

The base accelerogram of impact of the car with the barrier is used in all calculations. In case of head-on barrier with the velocity of 16 m/sec the accelerogram is shown on Fig. 8.1.

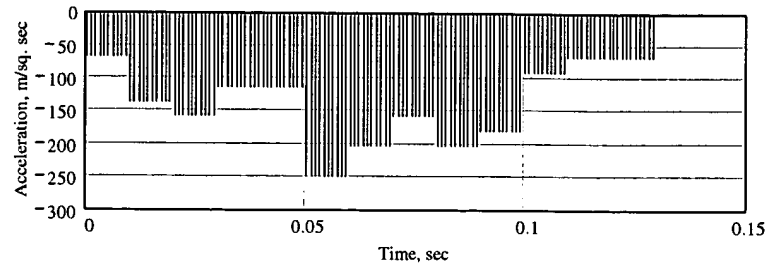


Fig. 8.1. The accelerogram of the car in case of head-on barrier with the velocity of 16 m/sec

On Fig. 8.2 the dependence of velocity of the car, corresponding to this accelerogram on time variation is exposed. As the diagram shows, the car obtains velocity of the opposite sign after impacting of the car with the barrier, in other words it bounces off the barrier.

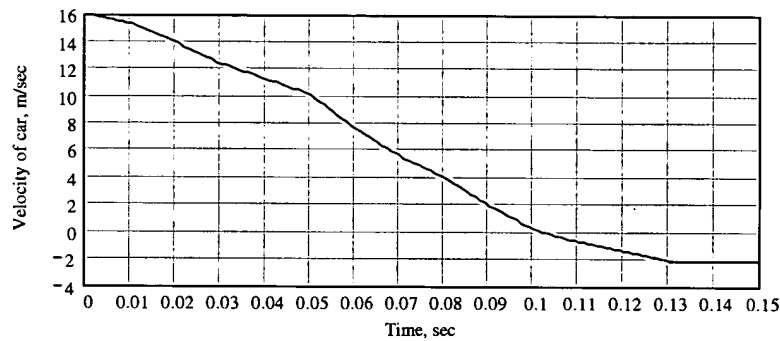


Fig. 8.2. Velocity of the car, corresponding to the accelerogram on time variation, exposed on Fig. 8.1

Location of the car is defined by the diagram, derived on Fig. 8.3. At that, a "braking distance" is accounted to 0.9 m.

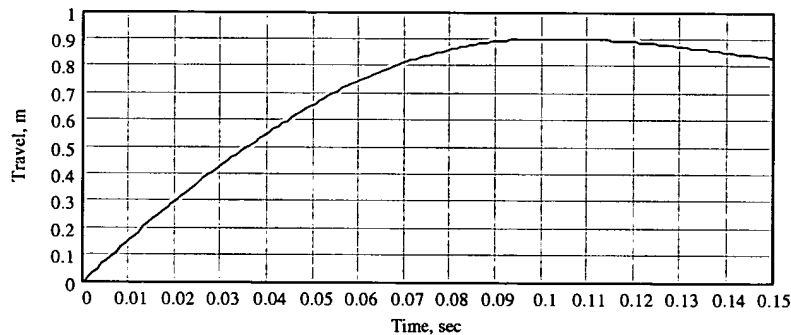


Fig. 8.3. Displacement of the car, corresponding to the accelerogram, exposed on Fig. 8.1

For other velocities and other cars the modified accelerograms are used.

Let the dependence of acceleration on time be expressed by a function  $a_{16}(t)$  in the base accelerogram. Then accelerograms for cars of other "rigidity", heading on barriers with other velocities, can be expressed by the formula:

$$(8.1) \quad a(k, v, t) = \frac{kv}{16 \text{ m/sec}} a_{16}(kt);$$

where  $k$  is a rigidity coefficient of the car;

$v$  is a velocity of head-on barrier of the car;

$t$  is time .

In this paper the cars with the rigidity coefficients of 1.0 or 1.3 were discussed. For example, the accelerogram for the “rigid” car at  $k=1.3$  is shown on Fig. 8.4.

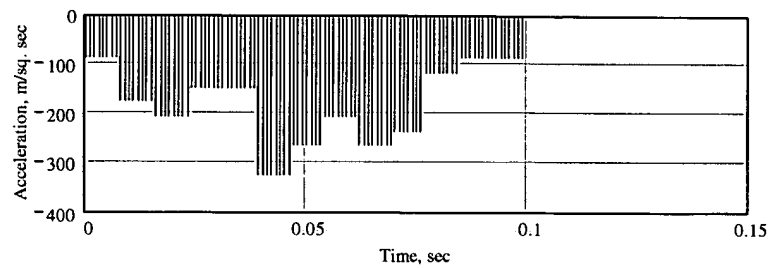


Fig. 8.4. The accelerogram for the “rigid” car ( $k=1.3$ ;  $v=16$  m/sec)

## 8.2. Formulation of solving problems

The solutions of the series of tasks on interaction among the car, the airbag and the occupant in case of impact of the car with the barrier are derived below.

The following initial data are similar for all tasks.

Data on the occupant:

- Mass of the occupant's trunk is 45 kg.
- “Rigidity” of the occupant's body is 500 kN/m.
- The initial location of the occupant is 0.4 m from the airbag bottom.

Data on the airbag:

- Thickness of the film is 0.1 mm;
- Modulus of elasticity of the film is 3600 MPa;
- Poisson's ratio of the film is 0.39;
- Gas within the airbag is nitrogen at the temperature of 100 degrees of Celsius;
- The overpressure at the final stage of inflation of the airbag is 14 kPa;
- Operate time of an indicator is 0.02 sec;
- Time of the airbag inflation with gas is 0.025 sec;
- Diameter of each of two holes for gas outflow is 3 sm.

Varying initial data:

- Diameter of the completely flattened airbag is 0.75 m and 0.625 m;
- Coefficient of rigidity of the car is 1.0 and 1.3;
- “Occupant's radius” if 0.8 m and 0.4 m;
- The initial velocities of the cars are 8 m/sec, 10 m/sec, 12 m/sec, 14 m/sec, 16 m/sec.

Thus, the quantity of the solved tasks is  $2 \times 2 \times 2 \times 5 = 40$ .

For each of the tasks the following parameters were defined as functions of time:

- Location of the front part of the occupant's trunk with respect to the bottom of the airbag;
- Variation of thickness of the airbag;
- Velocity of the center of gravity of the occupant with respect to the car;
- Factor of overloads of the occupant;
- Overpressure of gas within the airbag;
- Stresses within the bottom zone of the airbag;
- Stresses within the equator zone of the airbag;
- Stresses within the top zone of the airbag.

The range of time from 0 sec up to 0.15 sec was discussed.

The results of calculations are exposed as diagrams. There are  $40 \times 8 = 320$  diagrams. Besides, 20 diagrams, independent of the airbag and the occupant, were drawn.

### 8.3. The results, independent of the airbag and the occupant

Results of this type contain velocities and locations of the airbag's bottom in case of head-on the barrier.

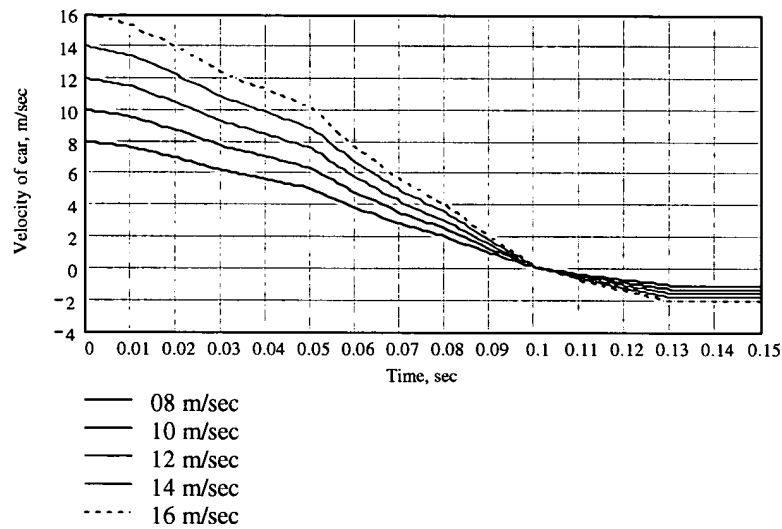


Fig. 8.5. Velocity of the airbag's bottom for the "soft" car ( $k=1$ )

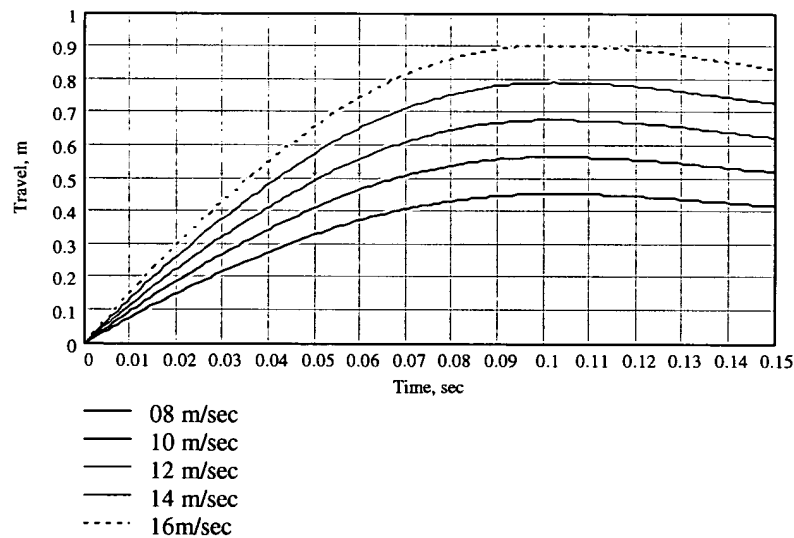


Fig. 8.6. Location of the airbag's bottom for the "soft" car ( $k=1$ )

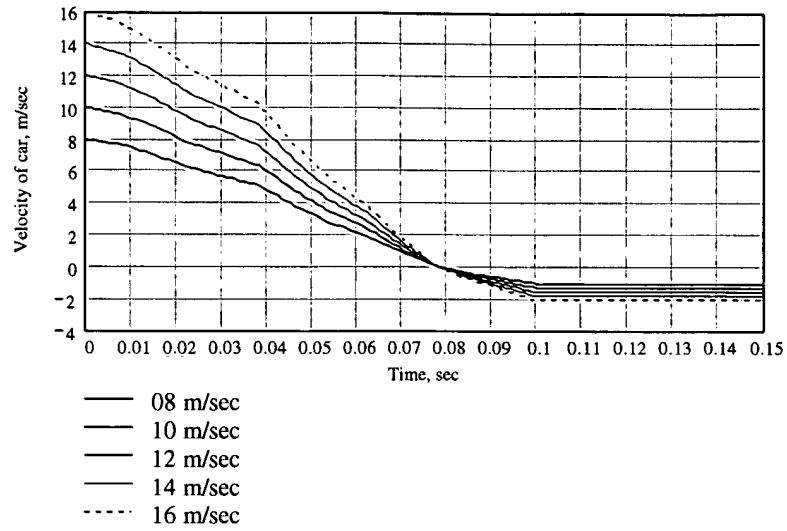


Fig. 8.7. Velocity of the airbag's bottom for the "rigid" car ( $k=1.3$ )

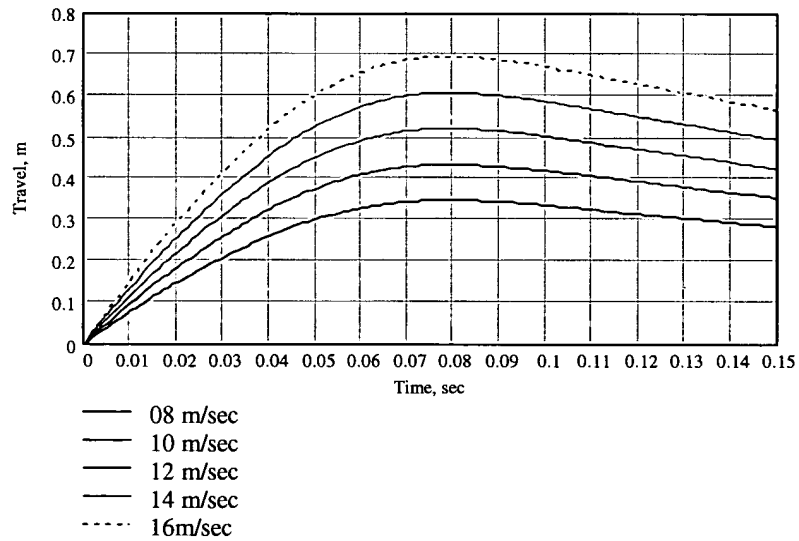


Fig. 8.8. Location of the airbag's bottom for the "rigid" car ( $k=1.3$ )

#### 8.4. The soft car with the airbag's diameter of 0.75 m and the occupant's radius of 0.8 m

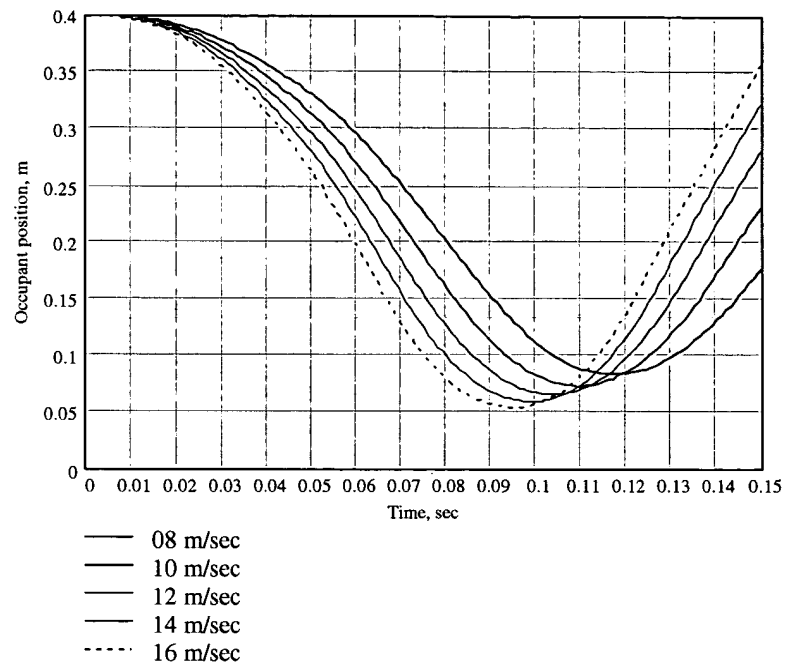


Fig. 8.9. Location of the front part of the occupant's trunk with respect to the airbag's bottom ( $d=0.75$  m;  $k=1$ ;  $R=0.8$  m)

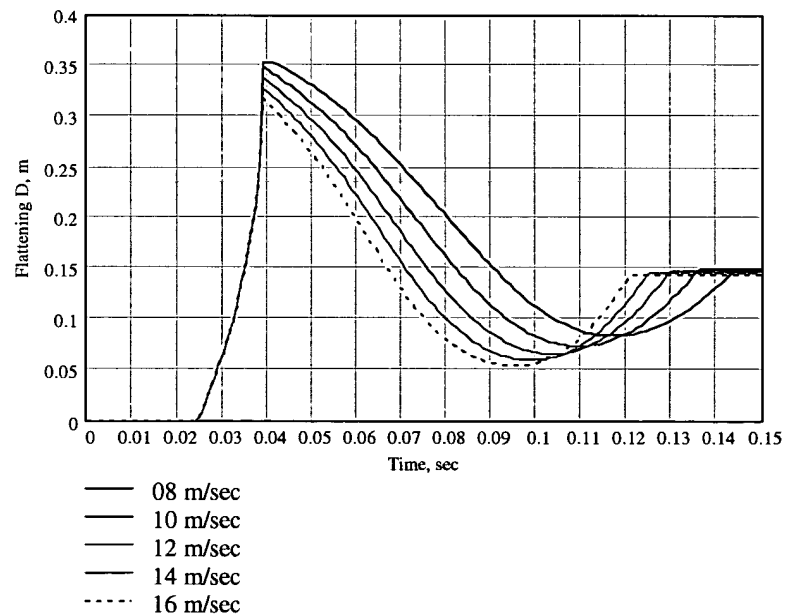


Fig. 8.10. Variation of thickness of the airbag  $D$  ( $d=0.75$  m;  $k=1$ ;  $R=0.8$  m)



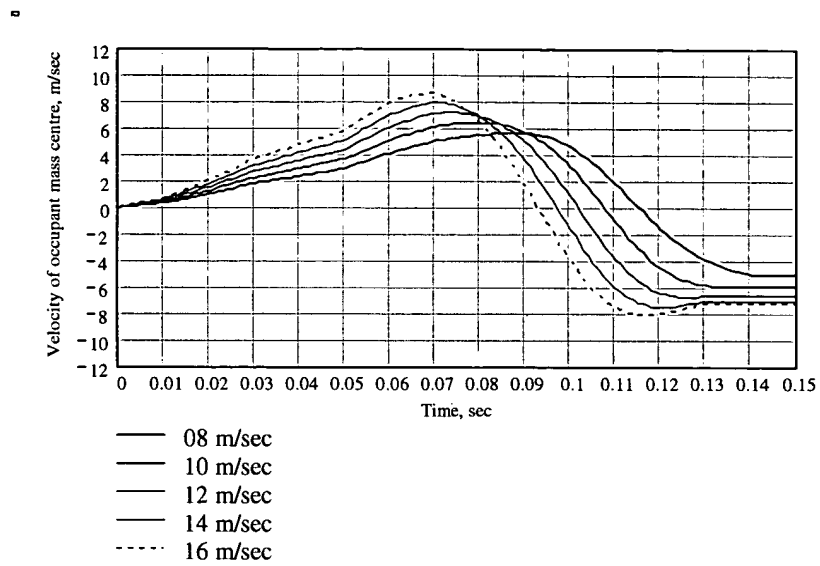


Fig. 8.11. Velocity of the occupant's center of gravity with respect to the car ( $d=0.75$  m;  $k=1$ ;  $R=0.8$  m)

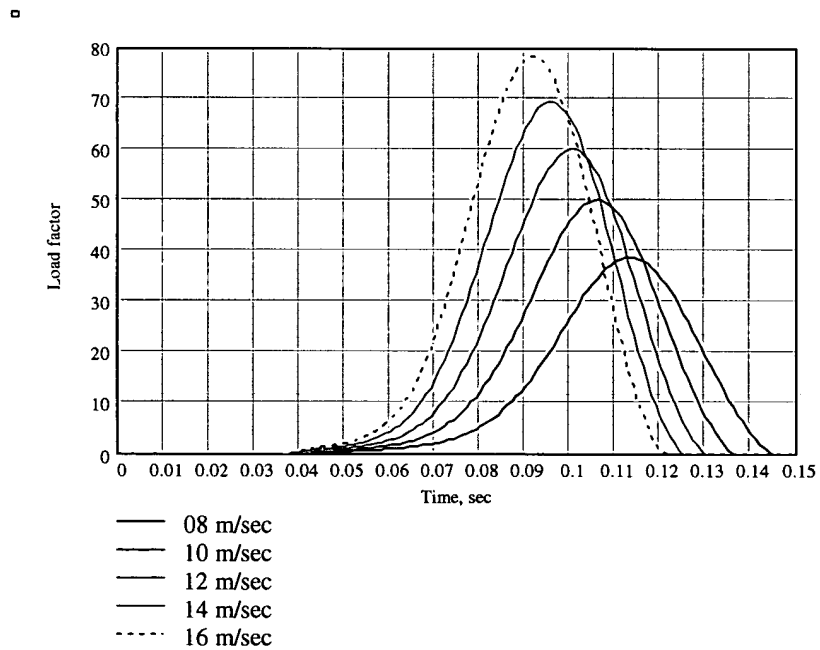


Fig. 8.12. Factor of overloads of the occupant ( $d=0.75$  m;  $k=1$ ;  $R=0.8$  m)

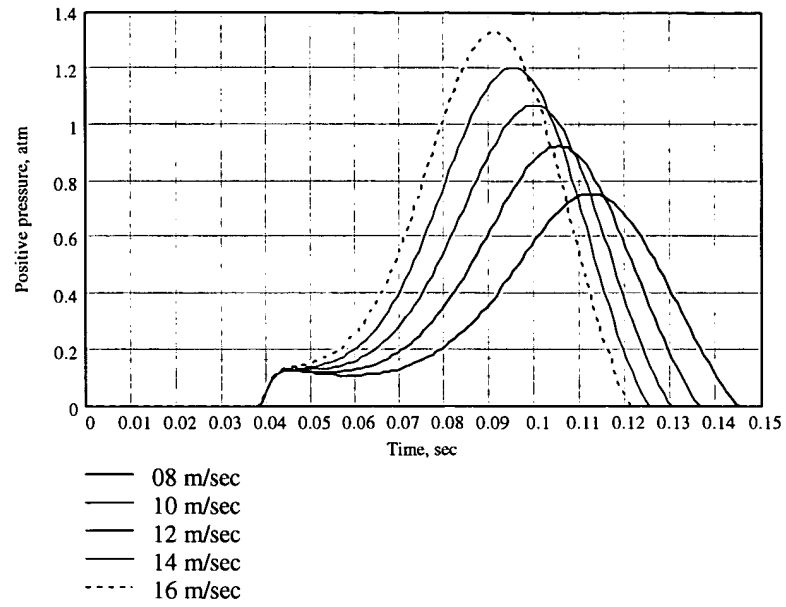


Fig. 8.13. Overpressure of gas within the airbag ( $d=0.75$  m;  $k=1$ ;  $R=0.8$  m)

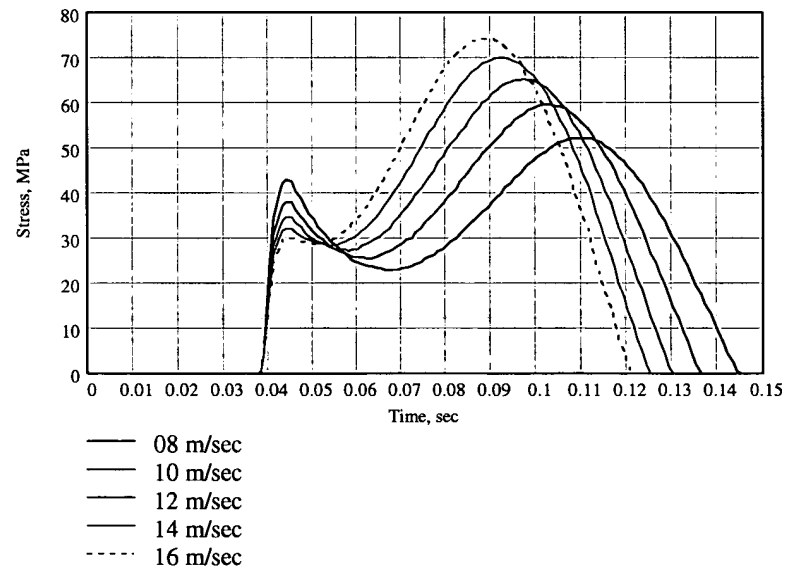


Fig. 8.14. Stresses within the airbag's bottom ( $d=0.75$  m;  $k=1$ ;  $R=0.8$  m)

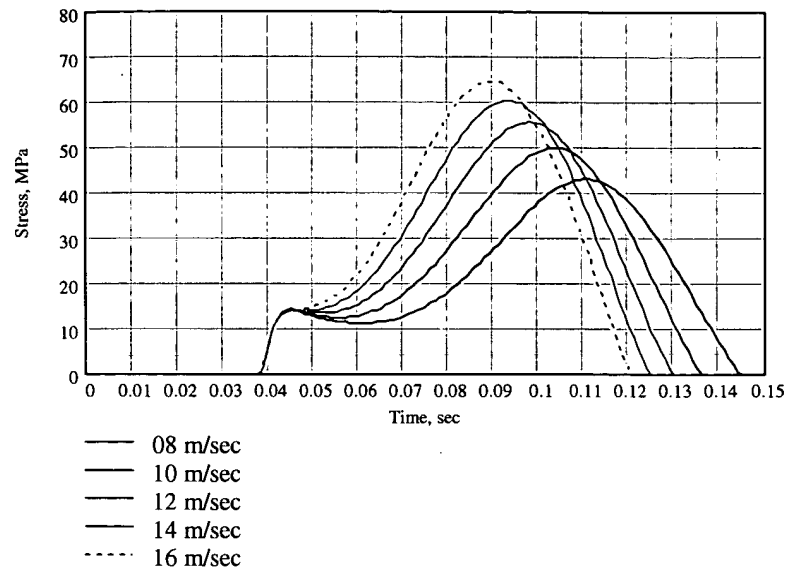


Fig. 8.15. Stresses within the equator zone of the airbag ( $d=0.75$  m;  $k=1$ ;  $R=0.8$  m)

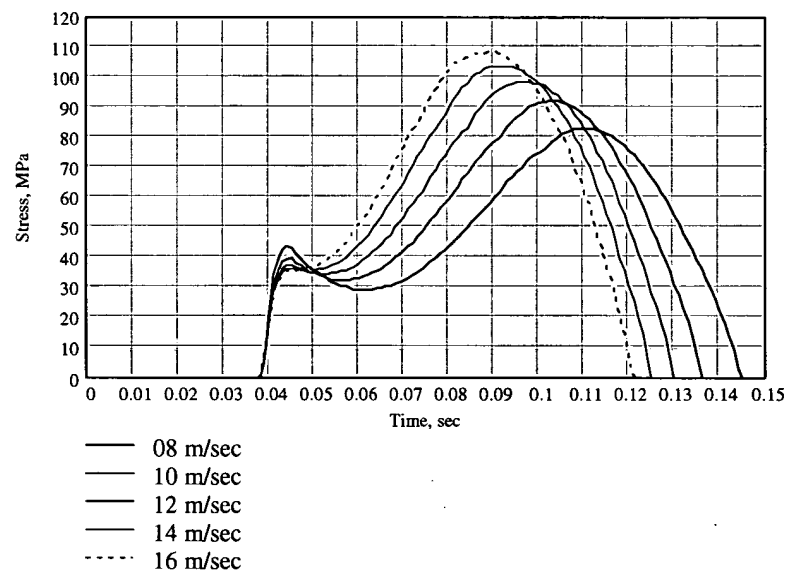


Fig. 8.16. Stresses in the top zone of the airbag ( $d=0.75$  m;  $k=1$ ;  $R=0.8$  m)

### 8.5. The soft car with the airbag's diameter of 0.75 m and the occupant's radius of 0.4 m

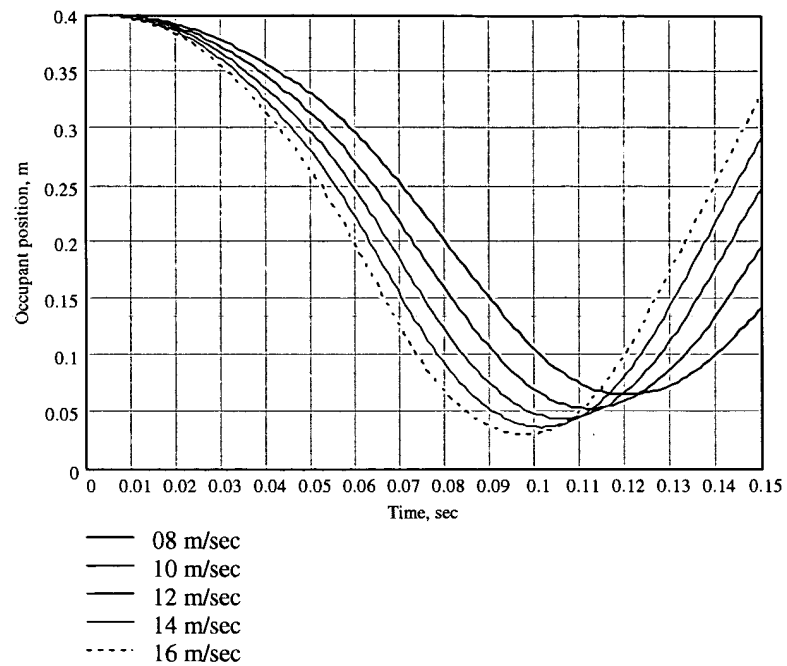


Fig. 8.17. Displacement of the front part of the occupant's trunk with respect to the airbag's bottom ( $d=0.75$  m;  $k=1$ ;  $R=0.4$  m)

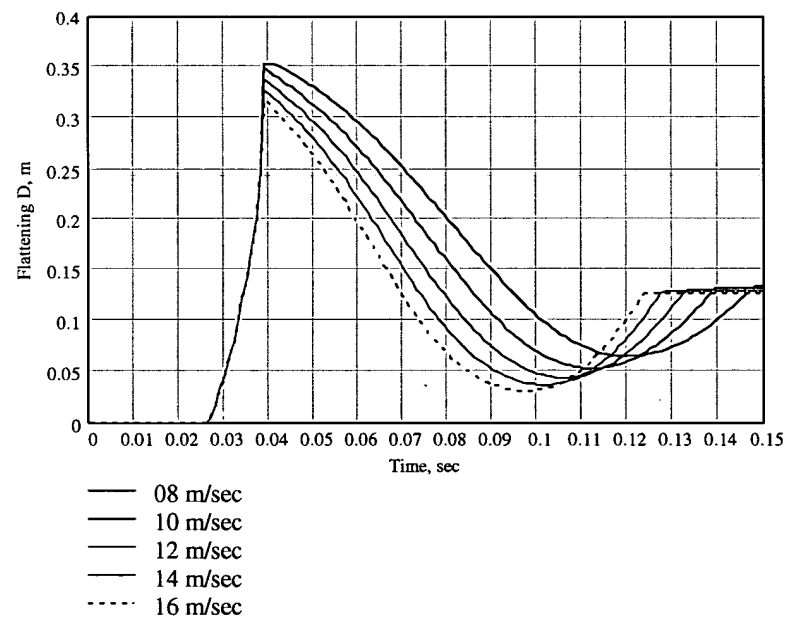


Fig. 8.18. Variation of thickness of the airbag  $D$  ( $d=0.75$  m;  $k=1$ ;  $R=0.4$  m)

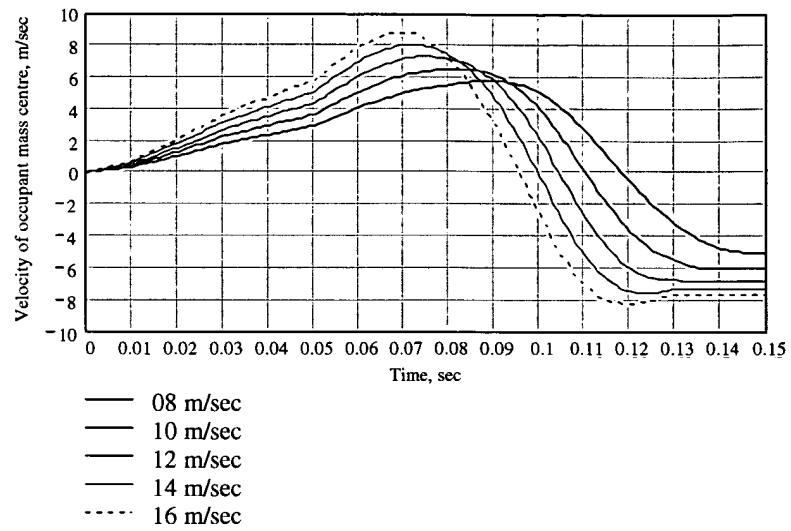


Fig. 8.19. Velocity of the occupant's center of gravity with respect to the car ( $d=0.75$  m;  $k=1$ ;  $R=0.4$  m)

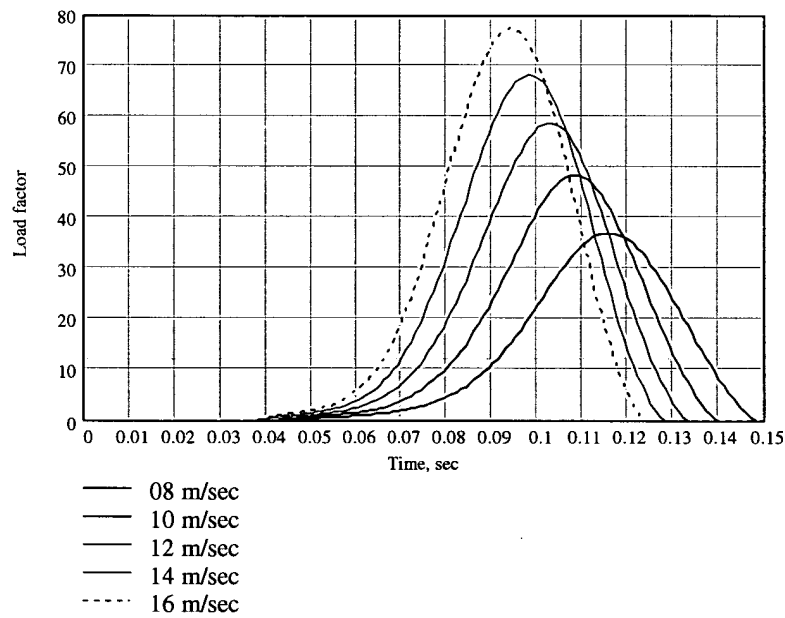


Fig. 8.20. Factor of overloads of the occupant ( $d=0.75$  m;  $k=1$ ;  $R=0.4$  m)

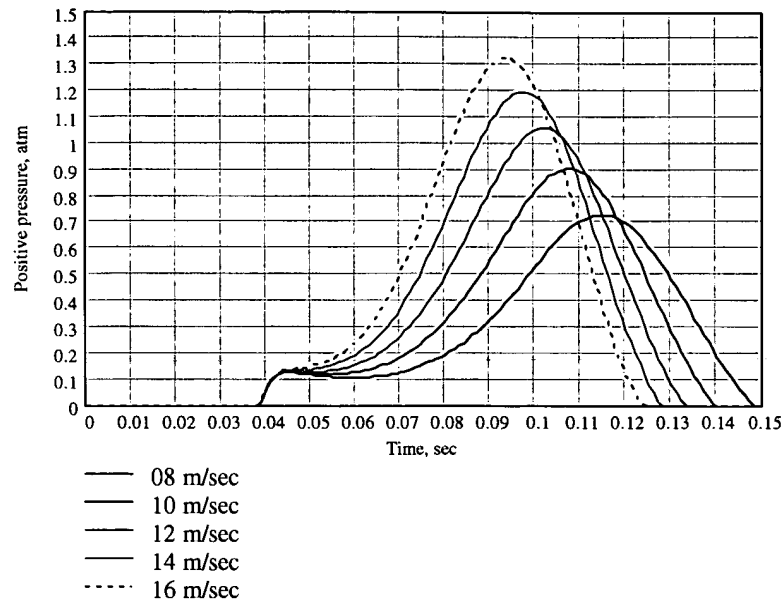


Fig. 8.21. Overload of gas within the airbag ( $d=0.75$  m;  $k=1$ ;  $R=0.4$  m)

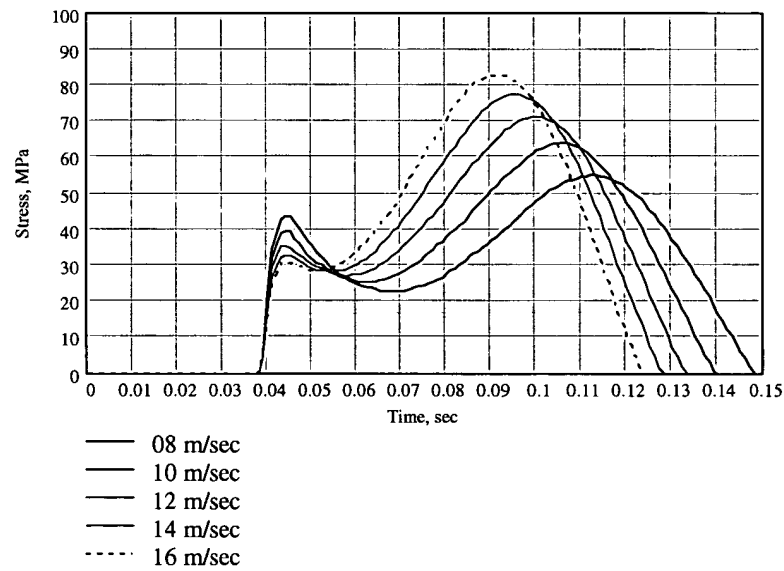


Fig. 8.22. Stresses within the bottom zone of the airbag ( $d=0.75$  m;  $k=1$ ;  $R=0.4$  m)

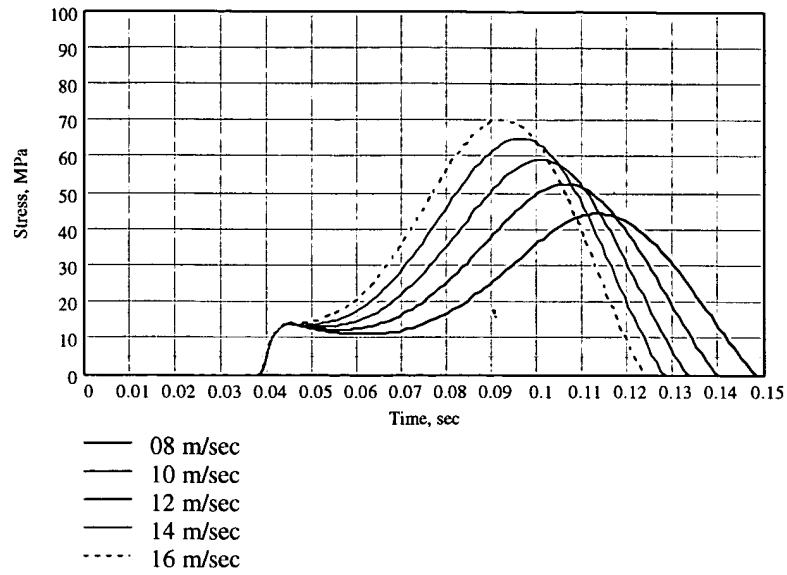


Fig. 8.23. Stresses within the equator zone of the airbag ( $d=0.75$  m;  $k=1$ ;  $R=0.4$  m)

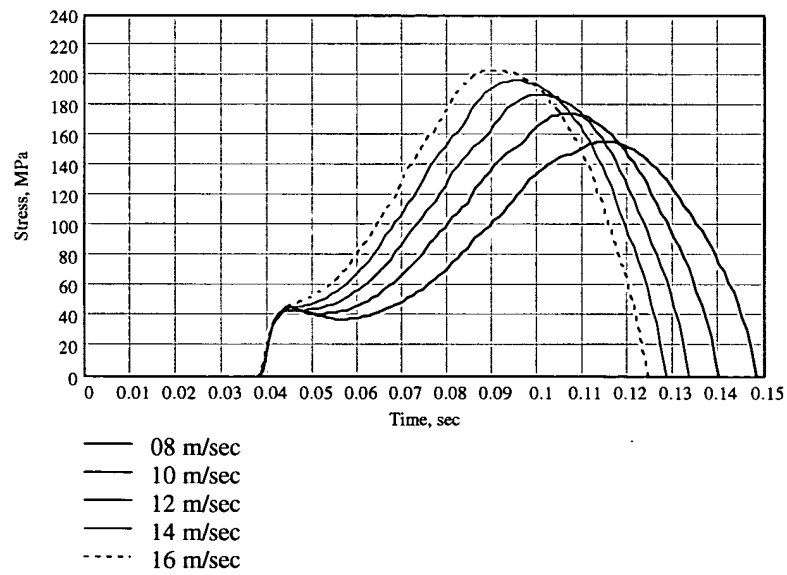


Fig. 8.24. Stresses within the top zone of the airbag ( $d=0.75$  m;  $k=1$ ;  $R=0.4$  m)

### 8.6. The rigid car with the airbag's diameter of 0.75 m and the occupant's radius of 0.8 m

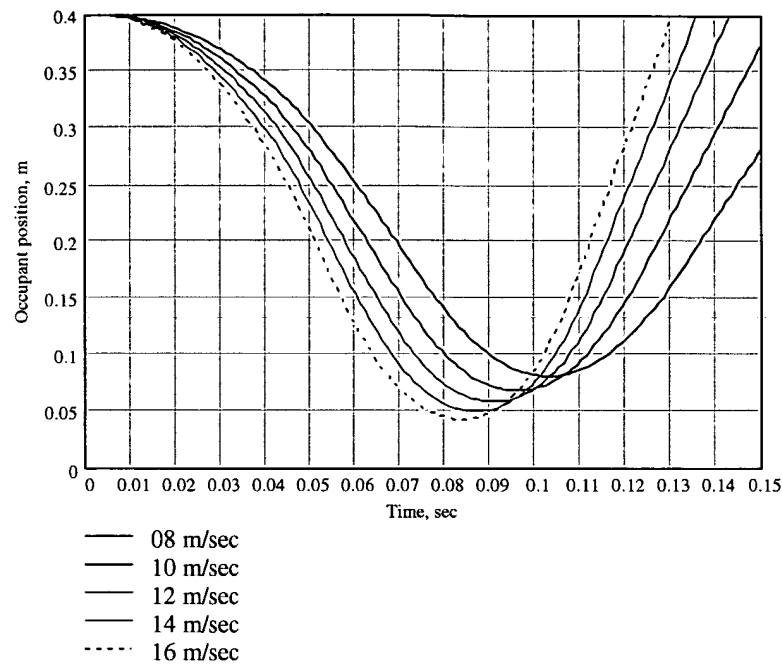


Fig. 8.25. Displacement of the front part of the occupant's trunk with respect to the airbag's bottom ( $d=0.75$  m;  $k=1.3$ ;  $R=0.8$  m)

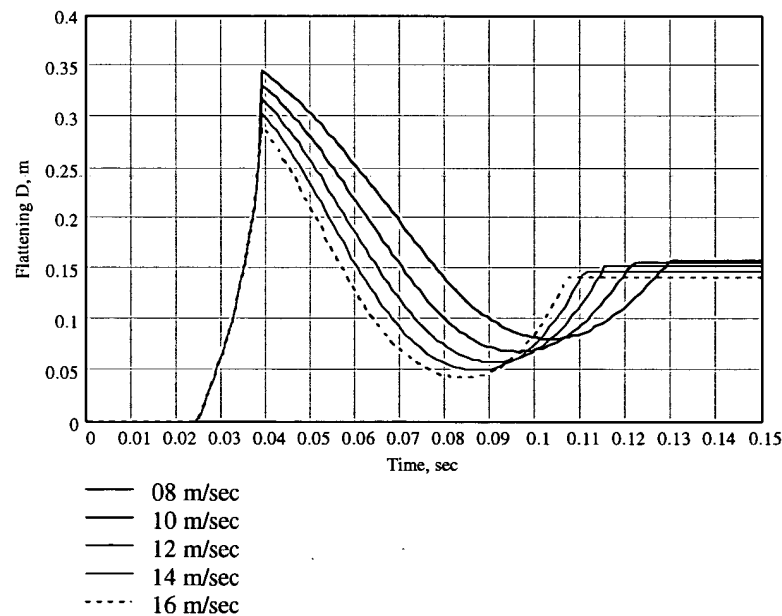


Fig. 8.26. Variation of thickness of the airbag  $D$  ( $d=0.75$  m;  $k=1.3$ ;  $R=0.8$  m)



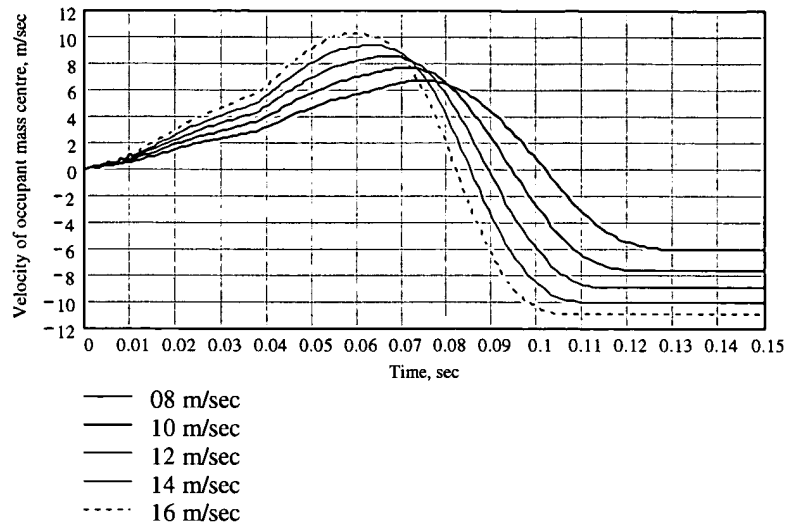


Fig. 8.27. Velocity of the occupant's center of gravity with respect to the car ( $d=0.75$  m;  $k=1.3$ ;  $R=0.8$  m)

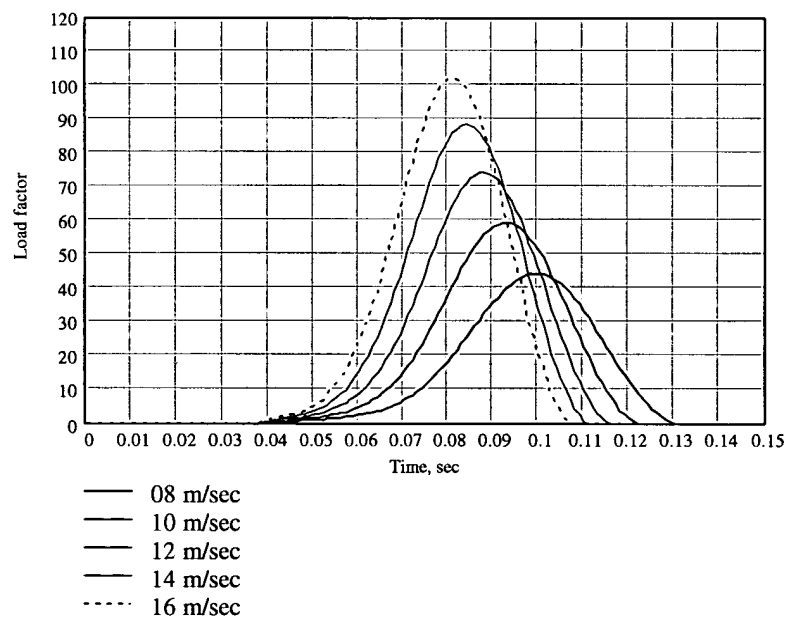


Fig. 8.28. Factor of overloads of the occupant ( $d=0.75$  m;  $k=1.3$ ;  $R=0.8$  m)

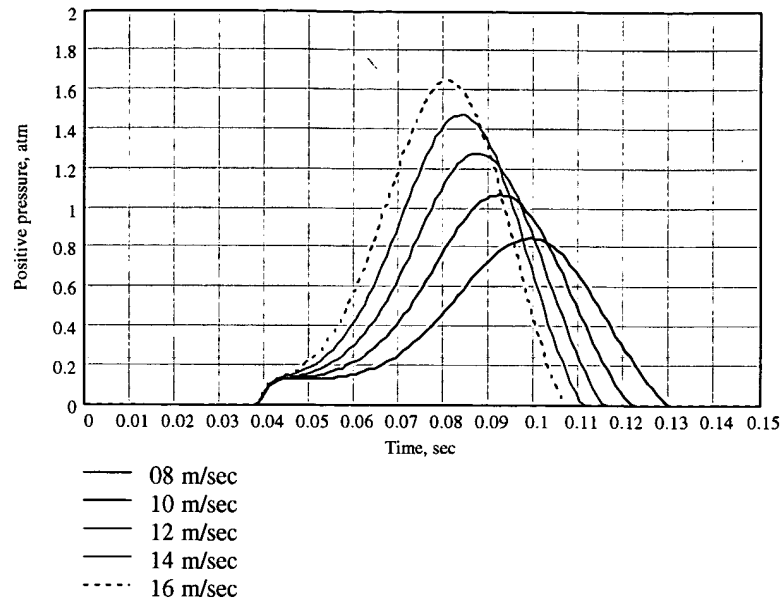


Fig. 8.29. Overpressure of gas within the airbag ( $d=0.75$  m;  $k=1.3$ ;  $R=0.8$  m)

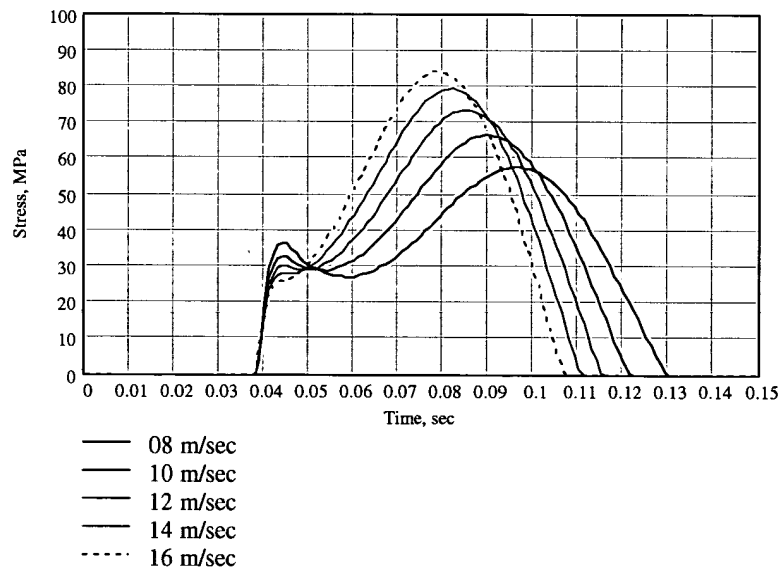


Fig. 8.30. Stresses within the bottom zone of the airbag ( $d=0.75$  m;  $k=1.3$ ;  $R=0.8$  m)

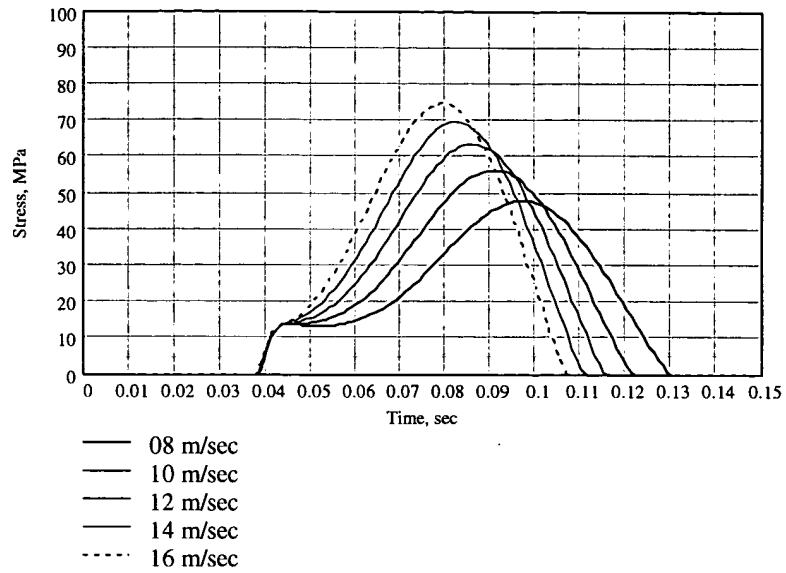


Fig. 8.31. Stresses within the equator zone of the airbag ( $d=0.75$  m;  $k=1.3$ ;  $R=0.8$  m)

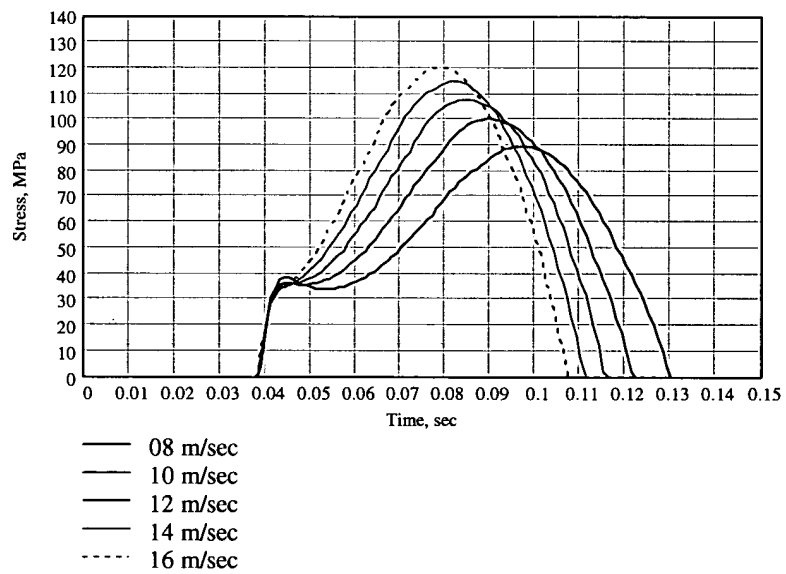


Fig. 8.32. Stresses within the top zone of the airbag ( $d=0.75$  m;  $k=1.3$ ;  $R=0.8$  m)

### 8.7. The rigid car with the airbag's diameter of 0.75 m and the occupant's radius of 0.4 m

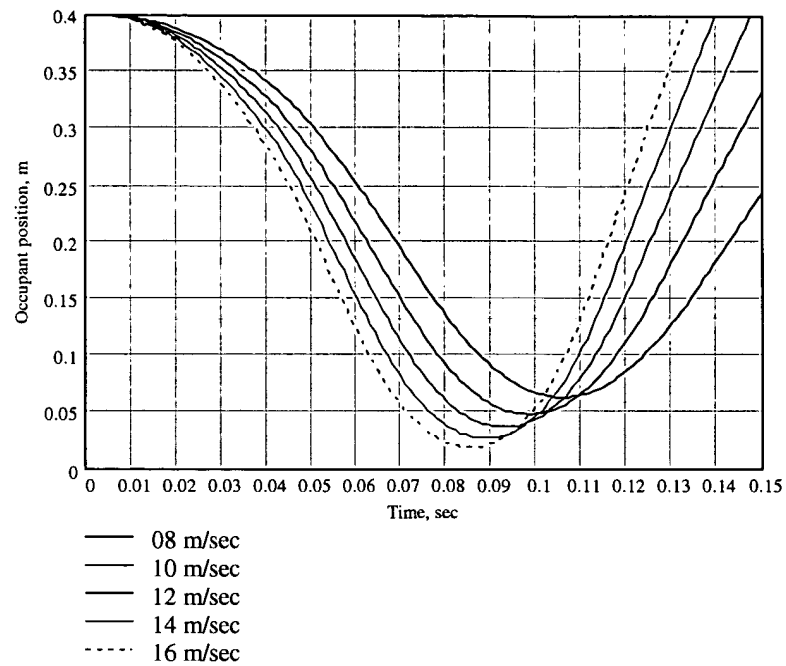


Fig. 8.33. Displacement of the front part of the occupant's trunk with respect to the bottom of the airbag ( $d=0.75$  m;  $k=1.3$ ;  $R=0.4$  m)

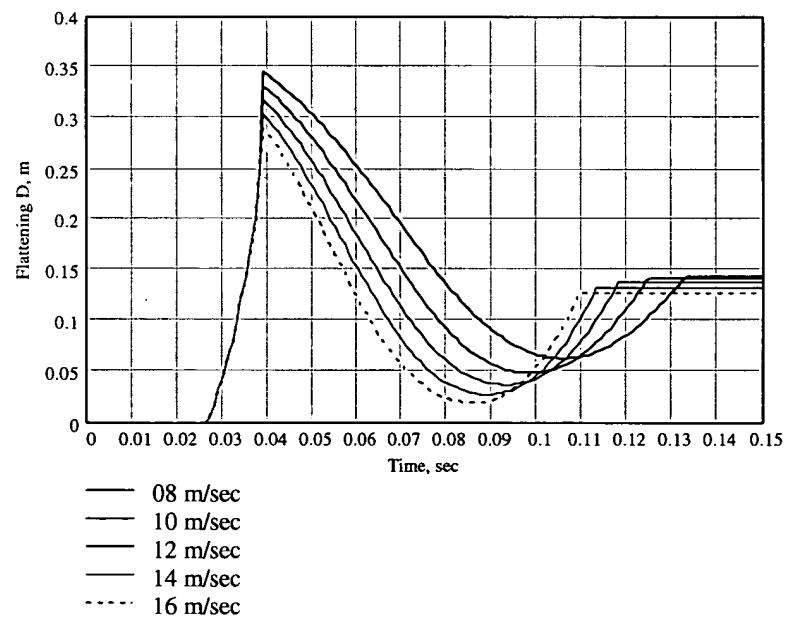


Fig. 8.34. Variation of thickness of the airbag  $D$  ( $d=0.75$  m;  $k=1.3$ ;  $R=0.4$  m)

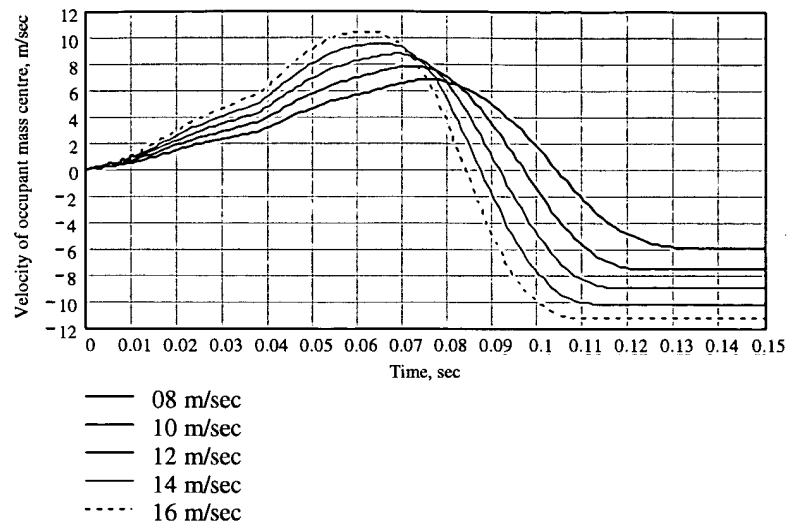


Fig. 8.35. Velocity of the occupant's center of gravity with respect to the car ( $d=0.75$  m;  $k=1.3$ ;  $R=0.4$  m)

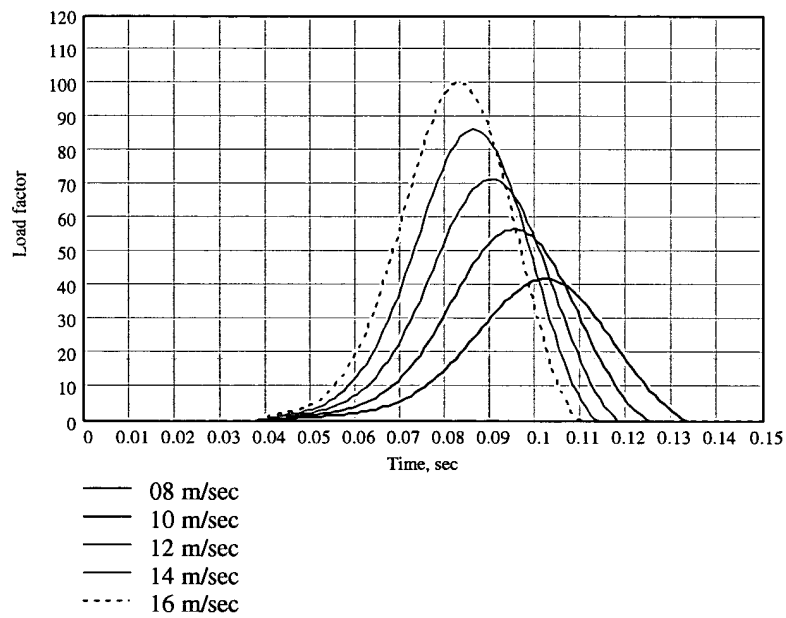


Fig. 8.36. Factor of overloads of the occupant ( $d=0.75$  m;  $k=1.3$ ;  $R=0.4$  m)

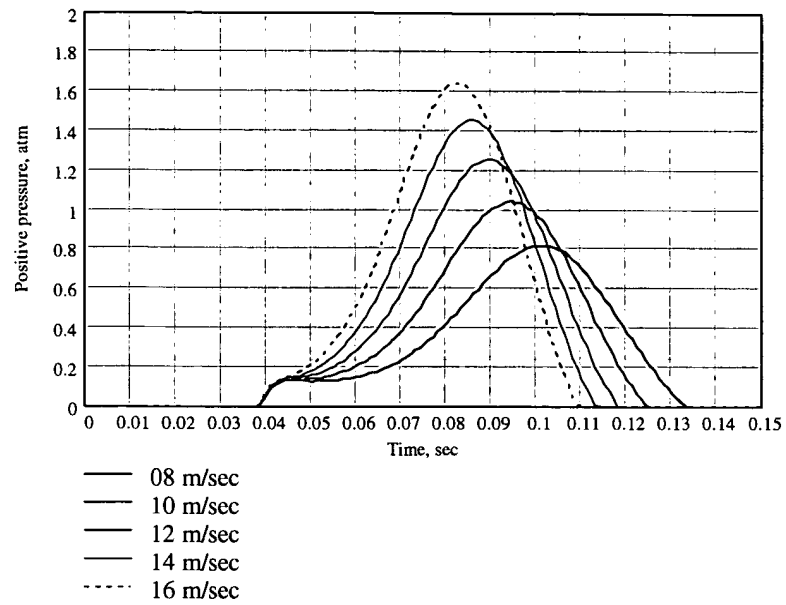


Fig. 8.37. Overpressure of gas within the airbag ( $d=0.75$  m;  $k=1.3$ ;  $R=0.4$  m)

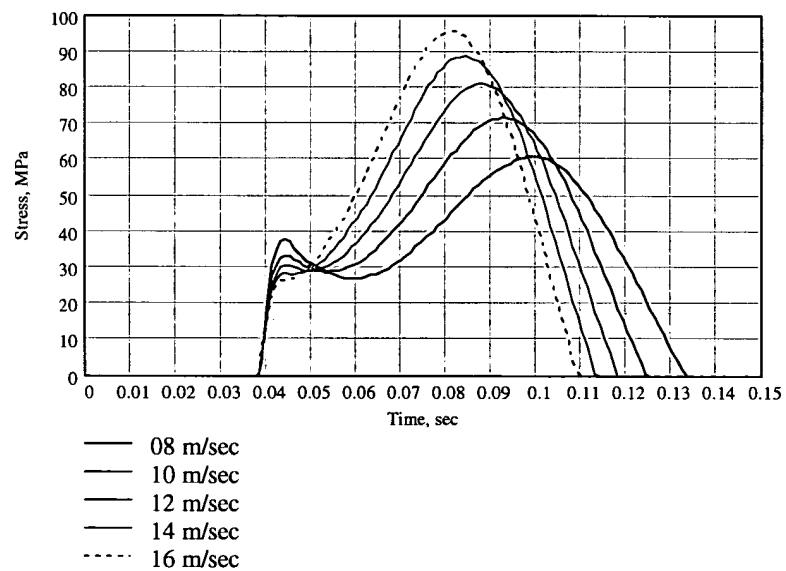


Fig. 8.38. Stresses within the bottom zone of the airbag ( $d=0.75$  m;  $k=1.3$ ;  $R=0.4$  m)

□

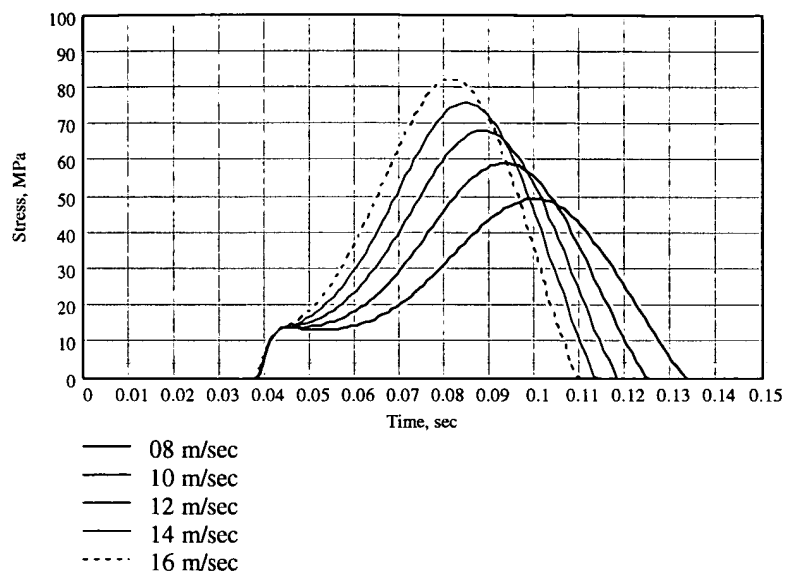


Fig. 8.39. Stresses within the equator zone of the airbag ( $d=0.75$  m;  $k=1.3$ ;  $R=0.4$  m)

□

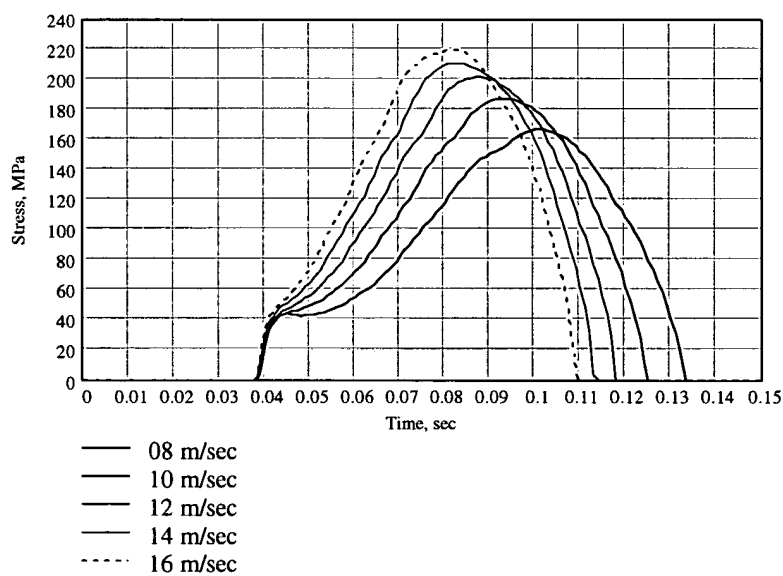


Fig. 8.40. Stresses within the top zone of the airbag ( $d=0.75$  m;  $k=1.3$ ;  $R=0.4$  m)

### 8.8. The soft car with the airbag's diameter of 0.625 m and the occupant's radius of 0.8 m

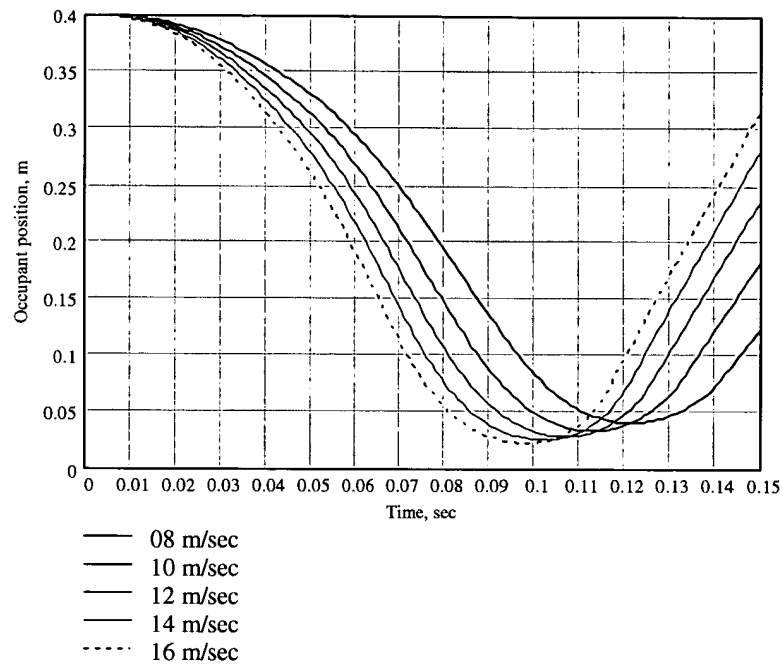


Fig. 8.41. Displacement of the front part of the occupant's trunk with respect to the bottom of the airbag ( $d=0.625$  m;  $k=1$ ;  $R=0.8$  m)

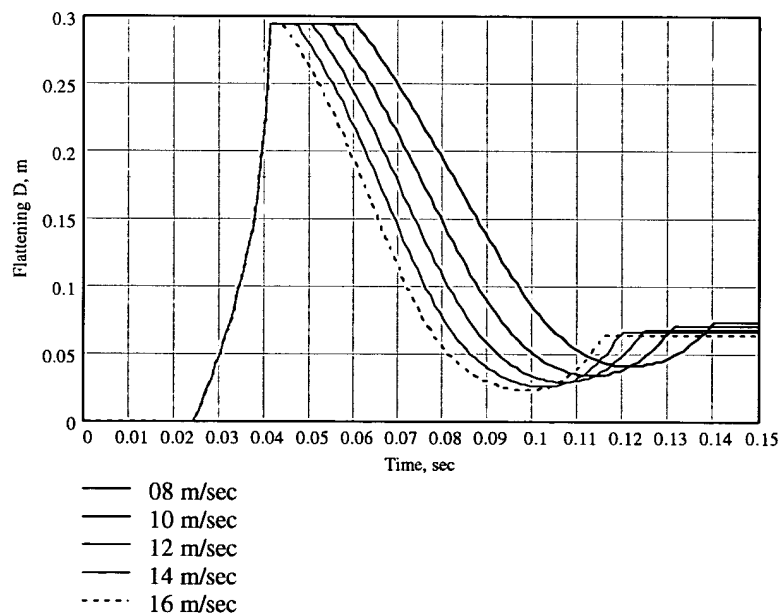


Fig. 8.42. Variation of thickness of the airbag  $D$  ( $d=0.625$  m;  $k=1$ ;  $R=0.8$  m)



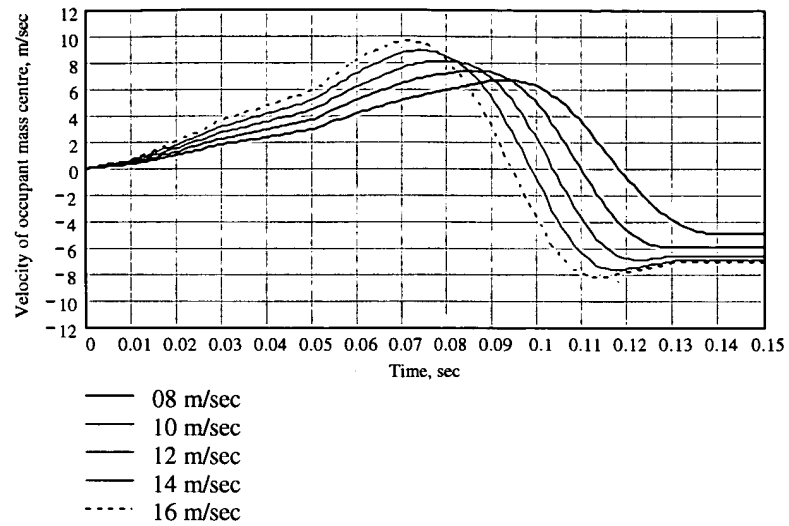


Fig. 8.43. Velocity of the occupant's center of gravity with respect to the car ( $d=0.625$  m;  $k=1$ ;  $R=0.8$  m)

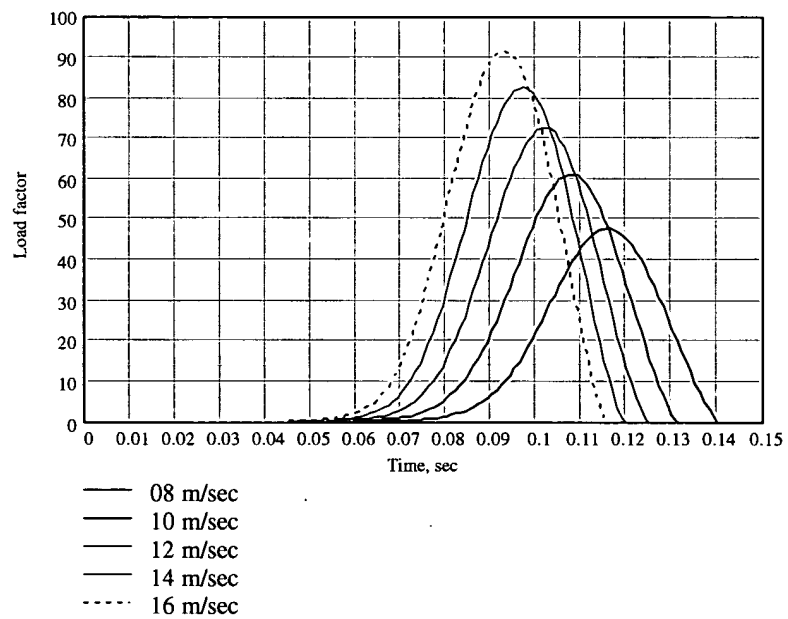


Fig. 8.44. Factor of overloads of the occupant ( $d=0.625$  m;  $k=1$ ;  $R=0.8$  m)

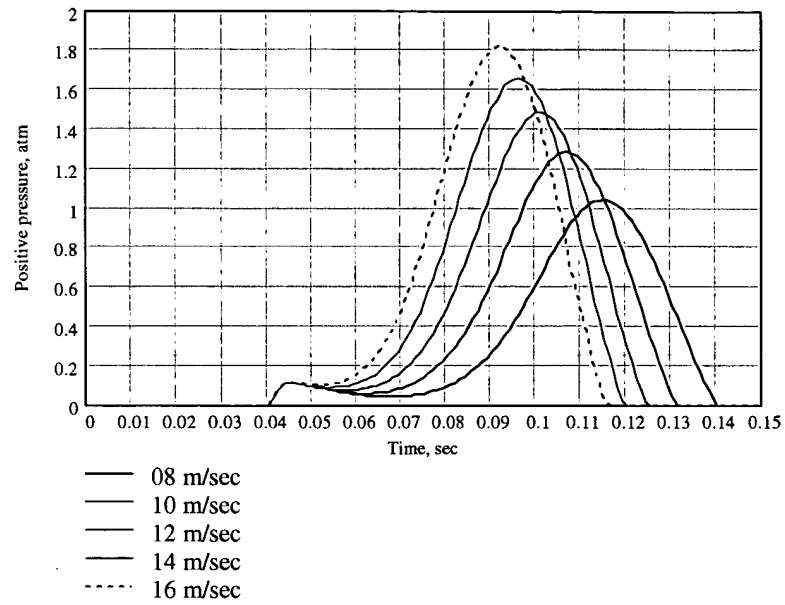


Fig. 8.45. Overpressure of gas within the airbag ( $d=0.625$  m;  $k=1$ ;  $R=0.8$  m)

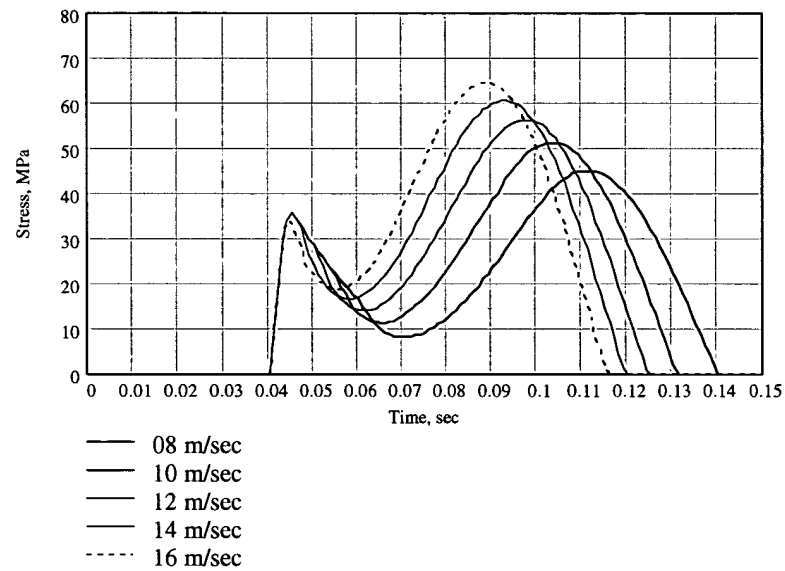


Fig. 8.46. Stresses in the bottom zone of the airbag ( $d=0.625$  m;  $k=1$ ;  $R=0.8$  m)

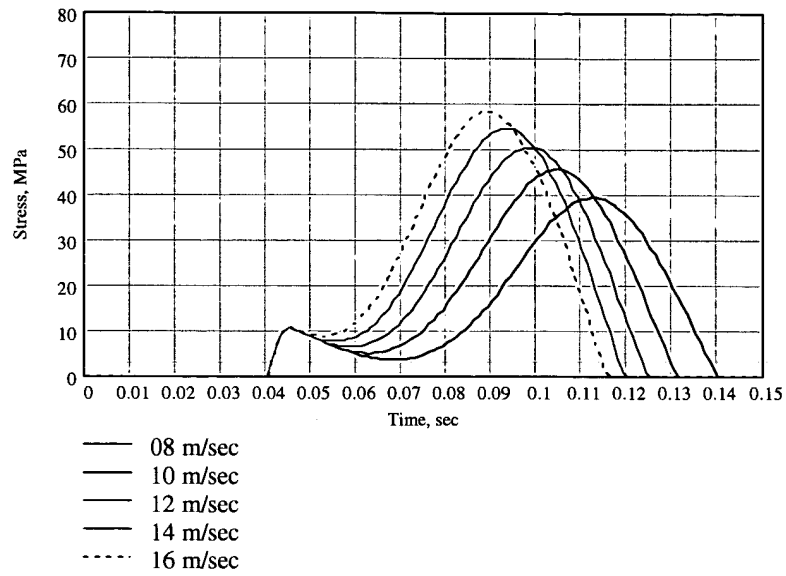


Fig. 8.47. Stresses in the equator zone of the airbag ( $d=0.625$  m;  $k=1$ ;  $R=0.8$  m)

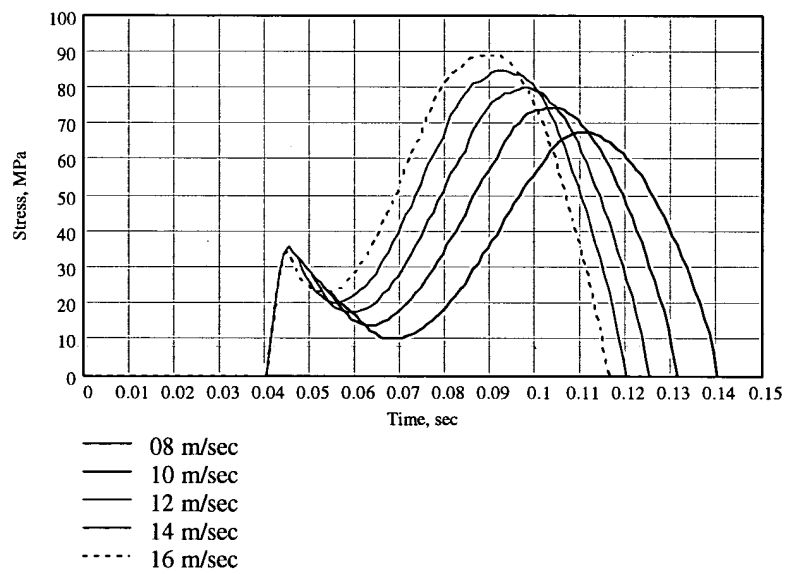


Fig. 8.48. Stresses in the top zone of the airbag ( $d=0.625$  m;  $k=1$ ;  $R=0.8$  m)

### 8.9. The soft car with the airbag's diameter of 0.625 m and the occupant's radius of 0.4 m

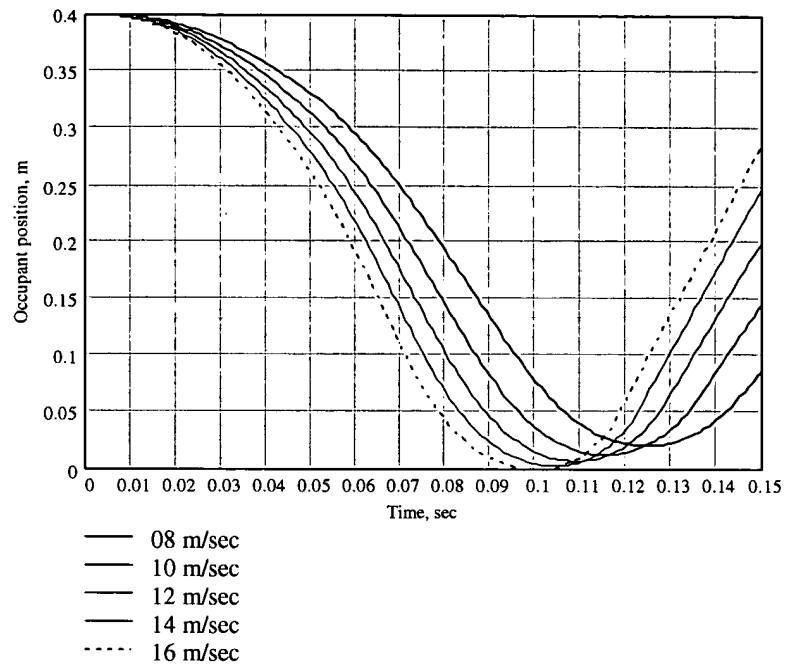


Fig. 8.49. Displacement of the front part of the occupant's trunk with respect to the airbag's bottom ( $d=0.625$  m;  $k=1$ ;  $R=0.4$  m)

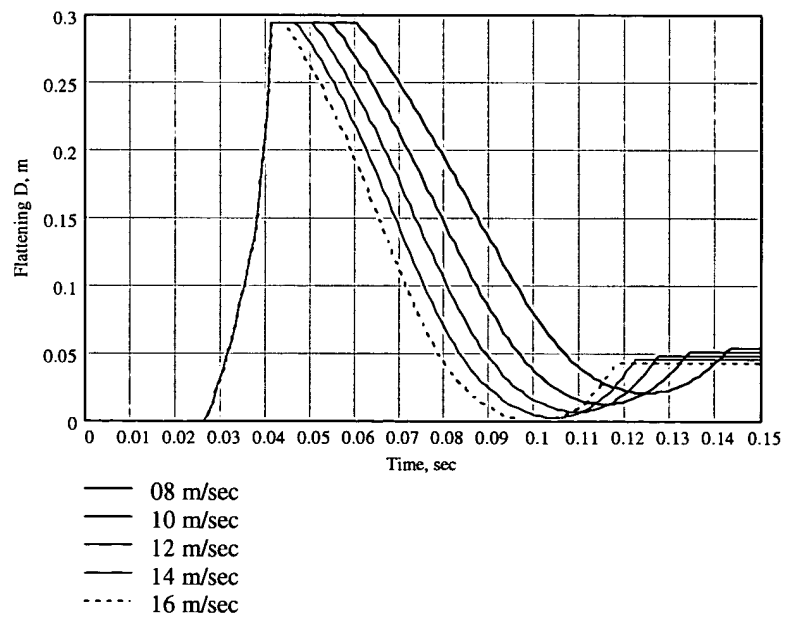


Fig. 8.50. Variation of thickness of the airbag  $D$  ( $d=0.625$  m;  $k=1$ ;  $R=0.4$  m)

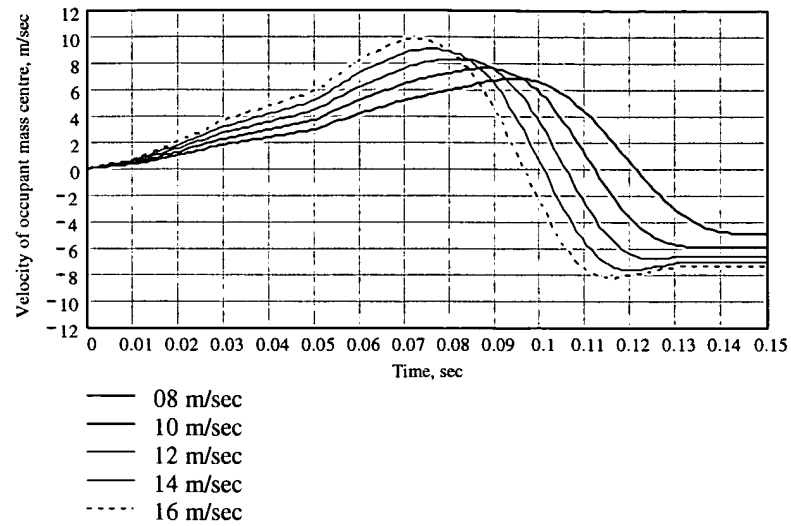


Fig. 8.51. Velocity of the occupant's center of gravity with respect to the car ( $d=0.625$  m;  $k=1$ ;  $R=0.4$  m)

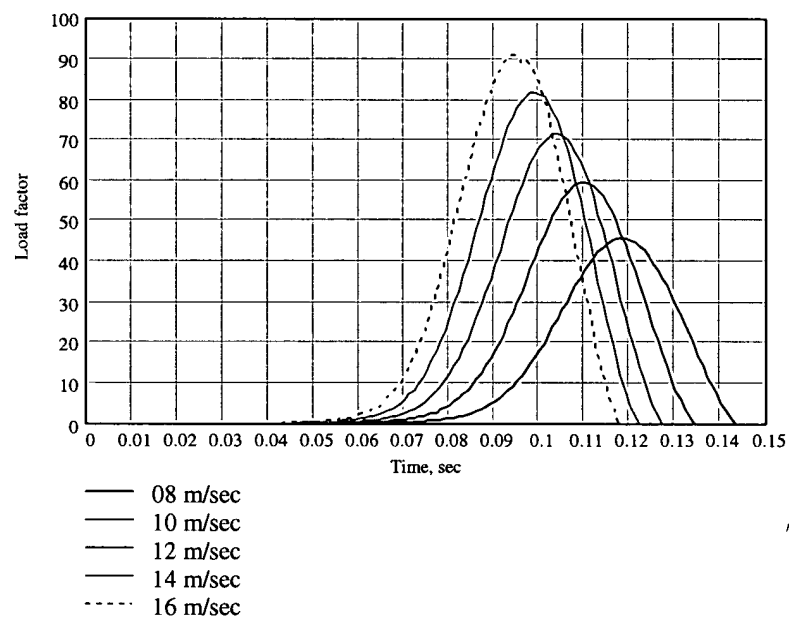


Fig. 8.52. Factor of overloads of the occupant ( $d=0.625$  m;  $k=1$ ;  $R=0.4$  m)

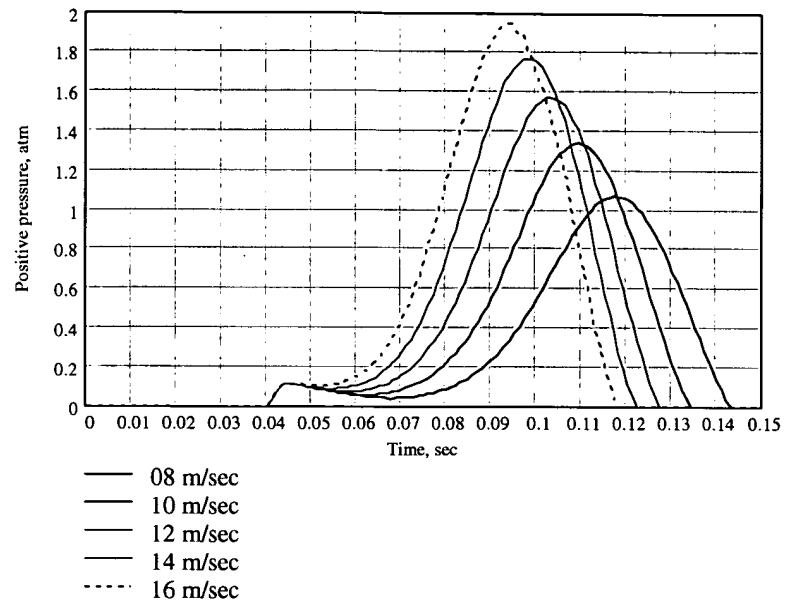


Fig. 8.53. Overpressure of gas within the airbag ( $d=0.625$  m;  $k=1$ ;  $R=0.4$  m)

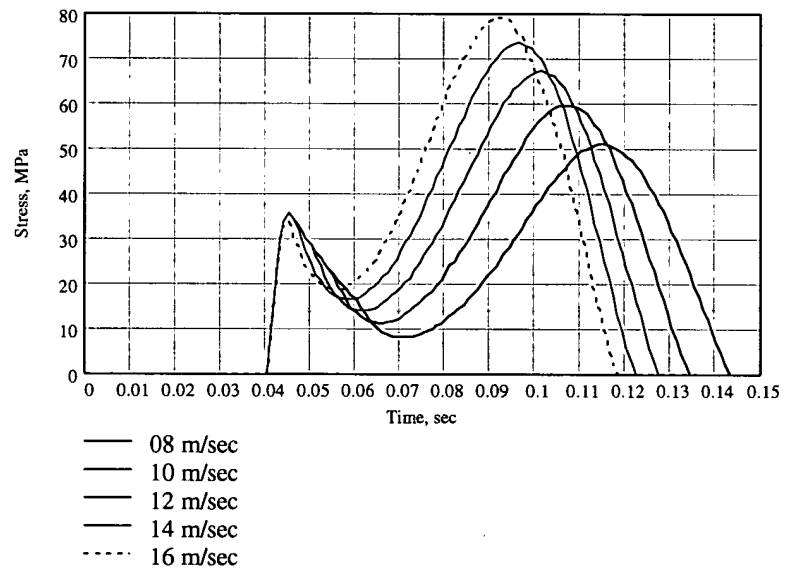


Fig. 8.54. Stresses within the bottom zone of the airbag ( $d=0.625$  m;  $k=1$ ;  $R=0.4$  m)

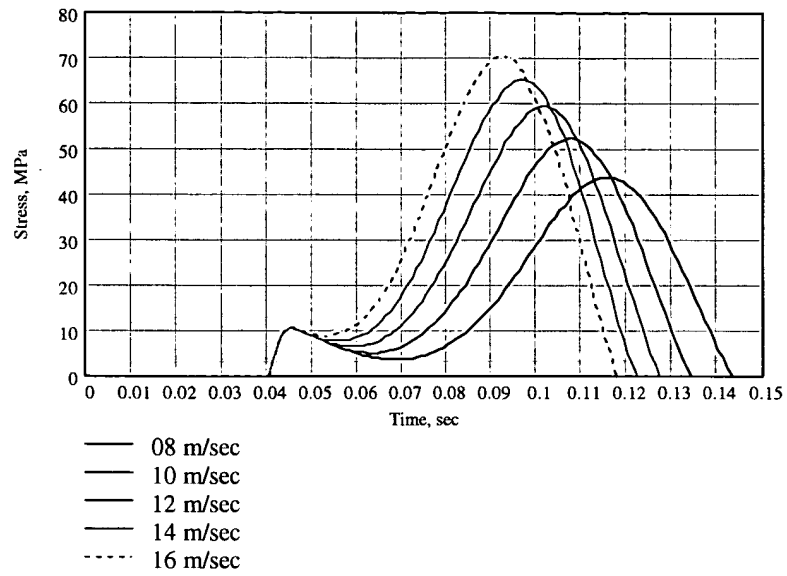


Fig. 8.55. Stresses within the equator zone of the airbag ( $d=0.625$  m;  $k=1$ ;  $R=0.4$  m)

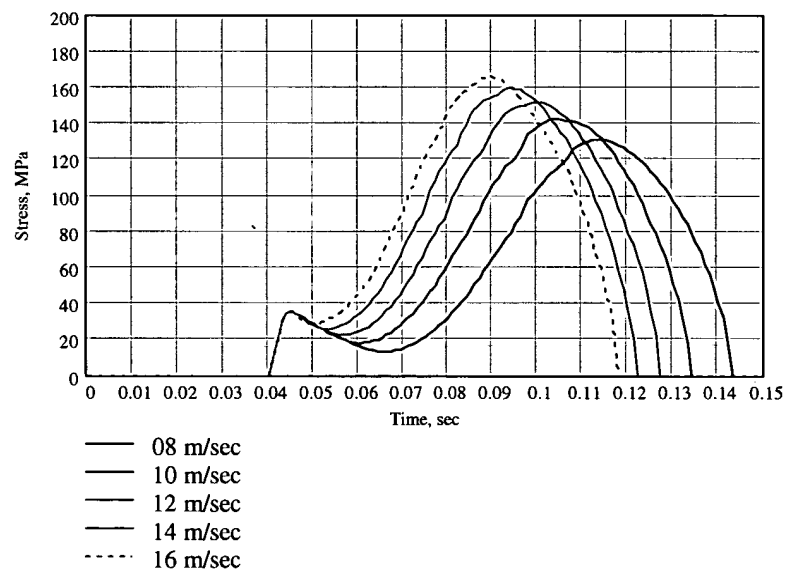


Fig. 8.56. Stresses within the top zone of the airbag ( $d=0.625$  m;  $k=1$ ;  $R=0.4$  m)

**8.10. The rigid car with the airbag's diameter of 0.625 m and the occupant's radius of 0.8 m**

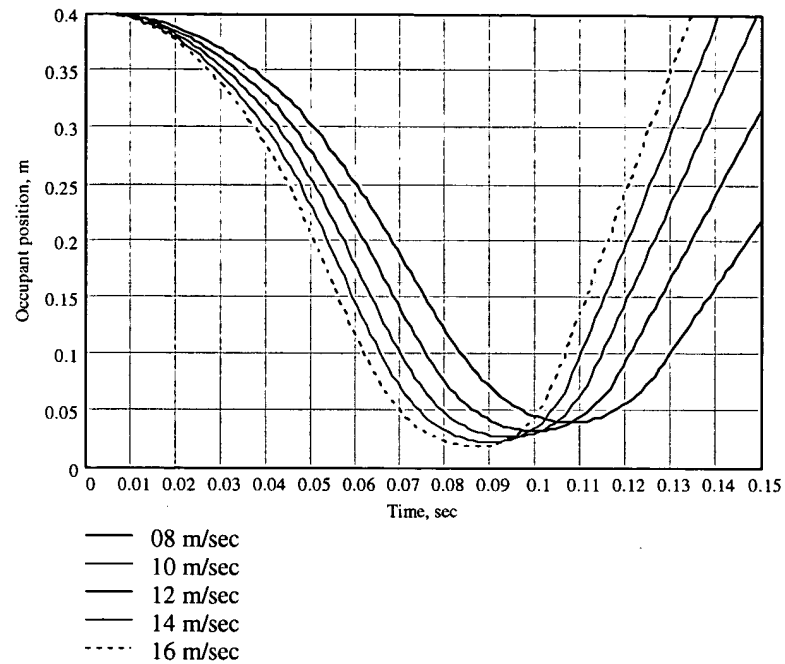


Fig. 8.57. Displacement of the front part of the occupant's trunk with respect to the airbag's bottom ( $d=0.625$  m;  $k=1.3$ ;  $R=0.8$  m)

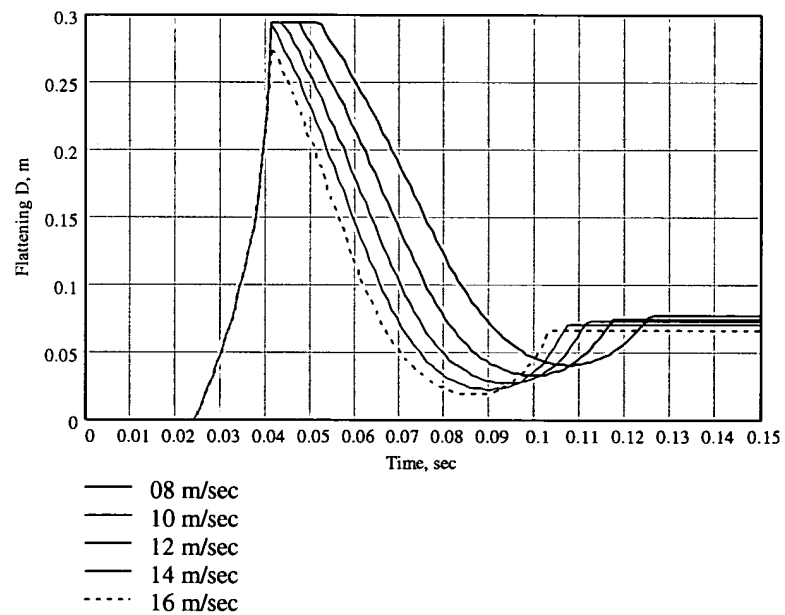


Fig. 8.58. Variation of thickness of the airbag  $D$  ( $d=0.625$  m;  $k=1.3$ ;  $R=0.8$  m)



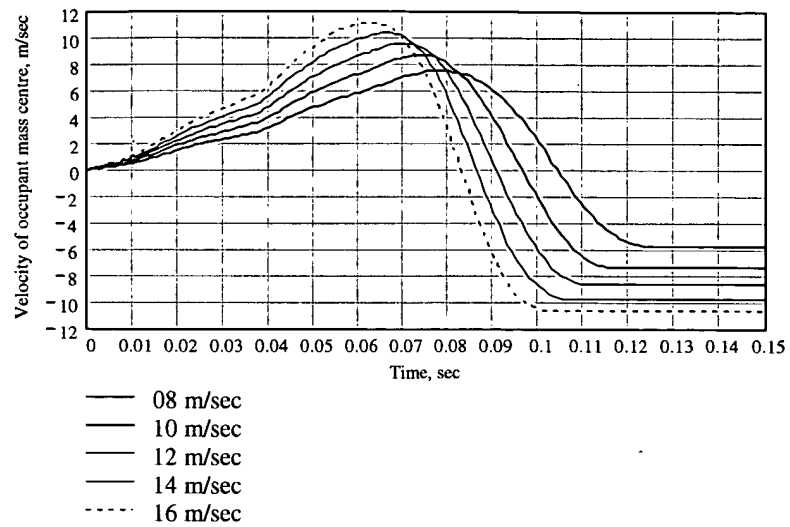


Fig. 8.59. Velocity of the occupant's center of gravity with respect to the car ( $d=0.625$  m;  $k=1.3$ ;  $R=0.8$  m)

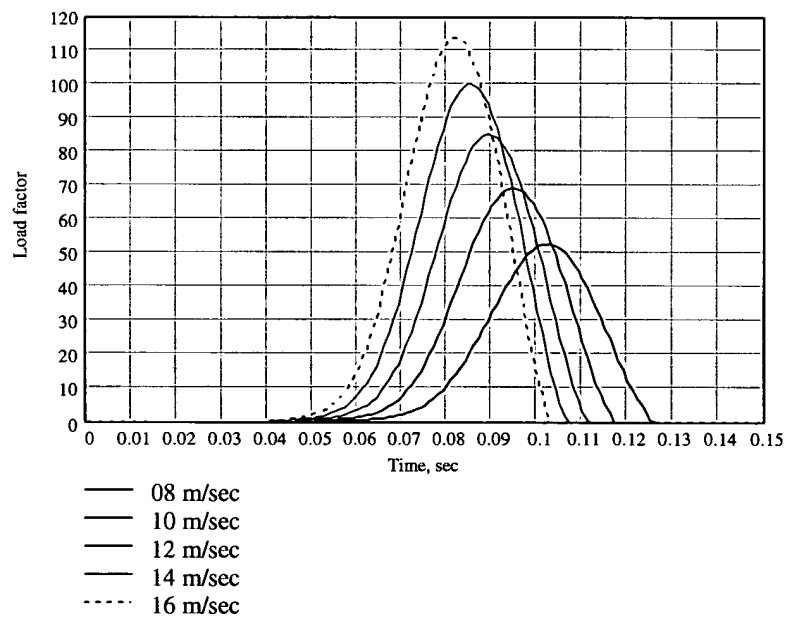


Fig. 8.60. Factor of overloads of the occupant ( $d=0.625$  m;  $k=1.3$ ;  $R=0.8$  m)

8

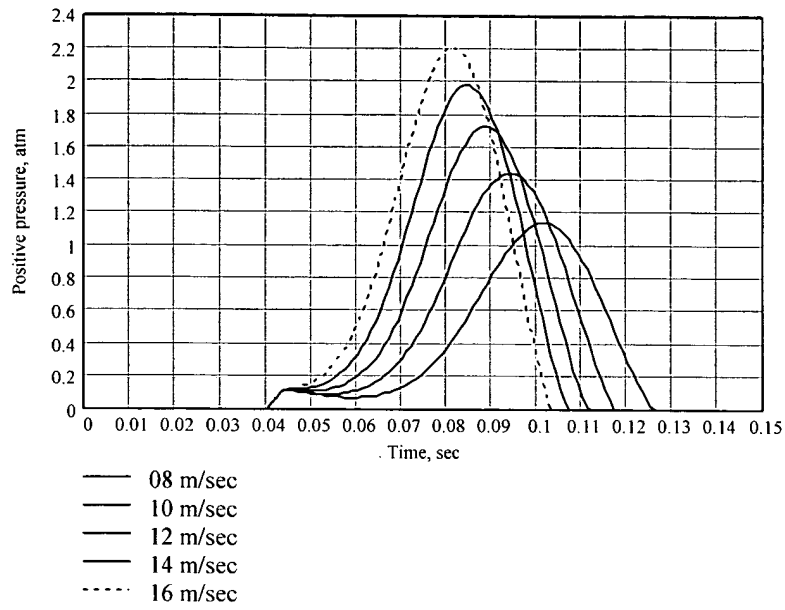


Fig. 8.61. Overpressure of gas within the airbag ( $d=0.625$  m;  $k=1.3$ ;  $R=0.8$  m)

82

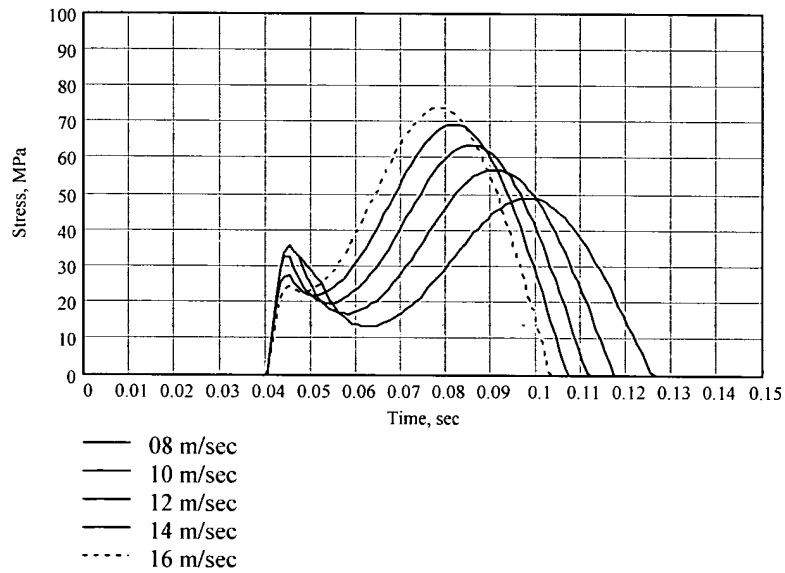


Fig. 8.62. Stresses within the bottom zone of the airbag ( $d=0.625$  m;  $k=1.3$ ;  $R=0.8$  m)

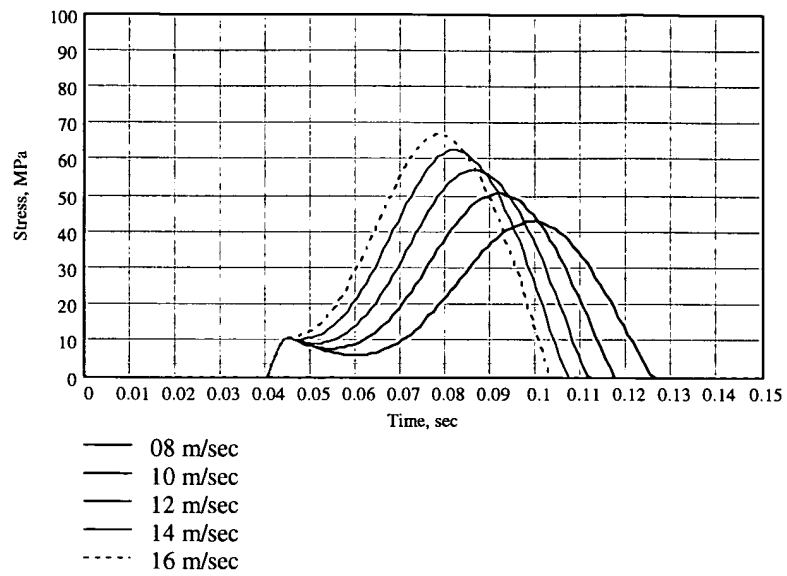


Fig. 8.63. Stresses within the equator zone of the airbag ( $d=0.625$  m;  $k=1.3$ ;  $R=0.8$  m)

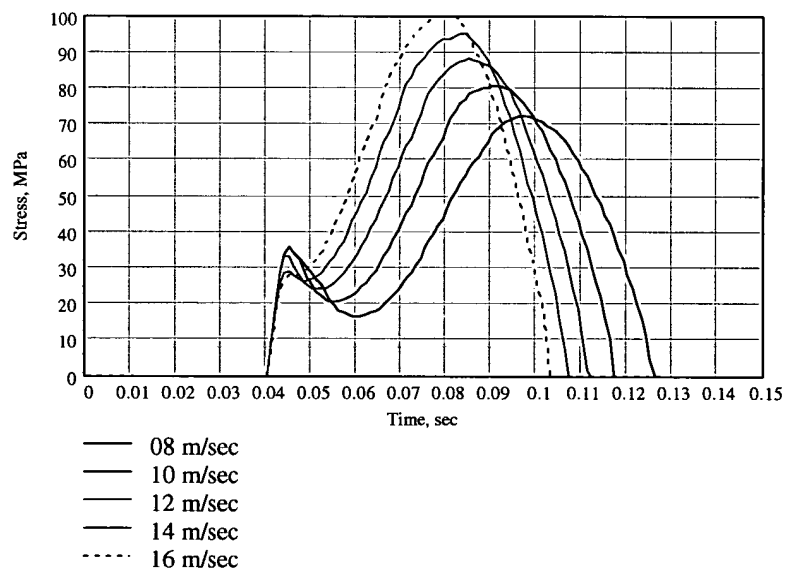


Fig. 8.64. Stresses within the top zone of the airbag ( $d=0.625$  m;  $k=1.3$ ;  $R=0.8$  m)

**8.11. The rigid car with the airbag's diameter of 0.625 m and the occupant's radius of 0.4 m**

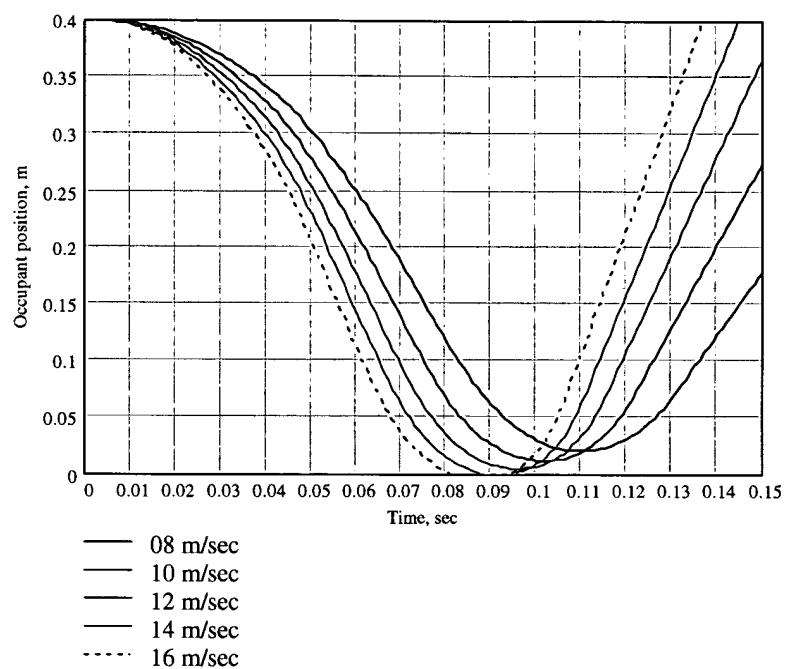


Fig. 8.65. Displacement of the front part of the occupant's trunk with respect to the airbag's bottom ( $d=0.625$  m;  $k=1.3$ ;  $R=0.4$  m)

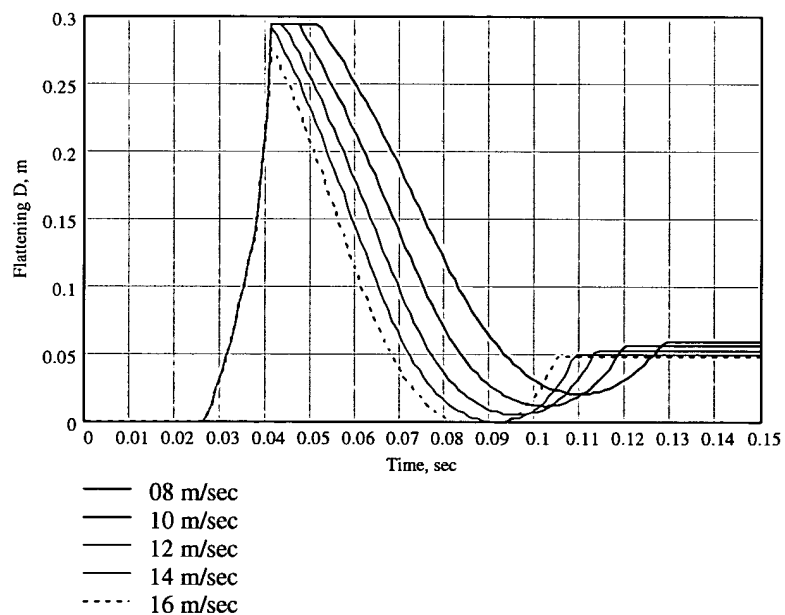


Fig. 8.66. Variation of thickness of the airbag  $D$  ( $d=0.625$  m;  $k=1.3$ ;  $R=0.4$  m)

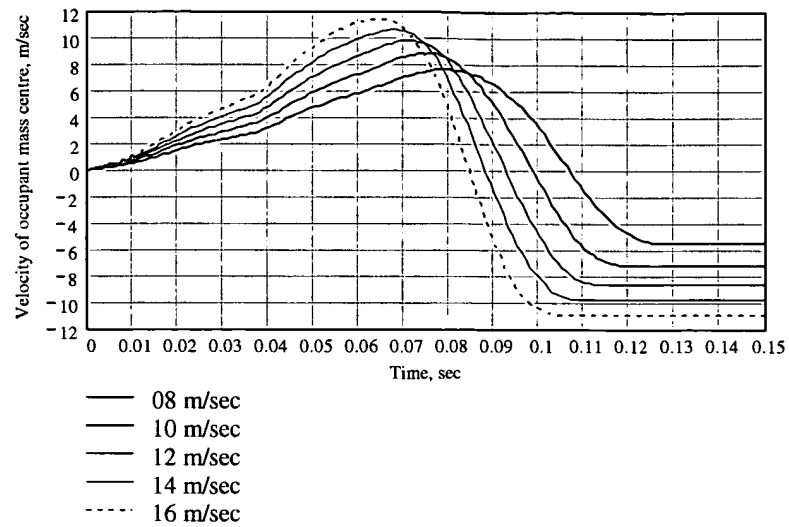


Fig. 8.67. Velocity of the occupant's center of gravity with respect to the car ( $d=0.625$  m;  $k=1.3$ ;  $R=0.4$  m)

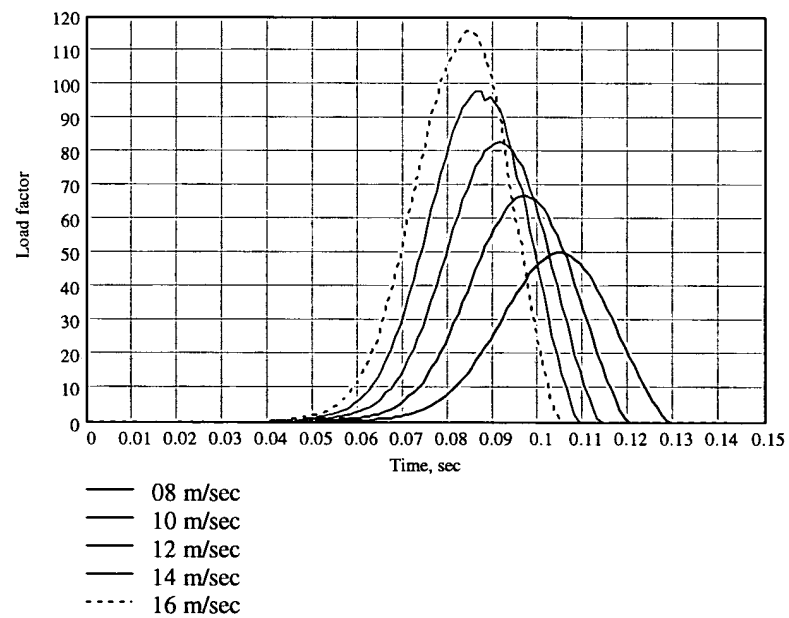


Fig. 8.68. Factor of overloads of the occupant ( $d=0.625$  m;  $k=1.3$ ;  $R=0.4$  m)

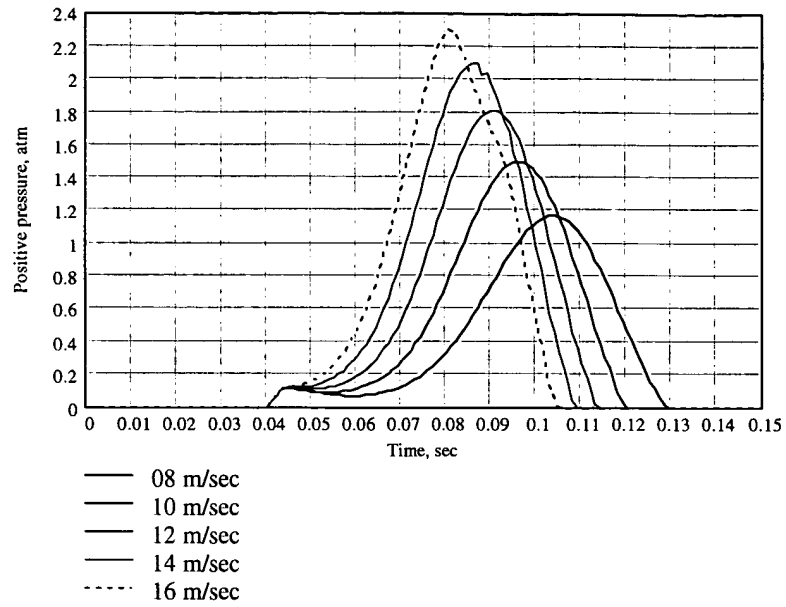


Fig. 8.69. Overpressure of gas within the airbag ( $d=0.625$  m;  $k=1.3$ ;  $R=0.4$  m)

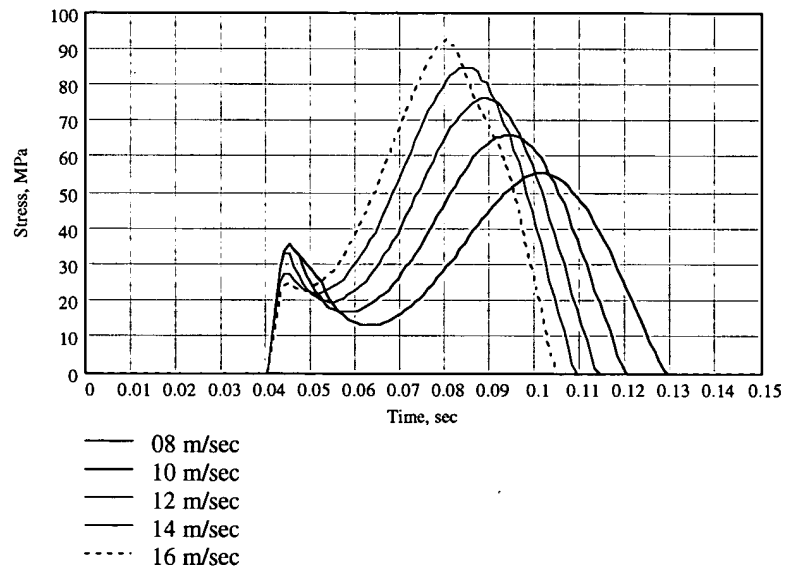


Fig. 8.70. Stresses within the bottom zone of the airbag ( $d=0.625$  m;  $k=1.3$ ;  $R=0.4$  m)

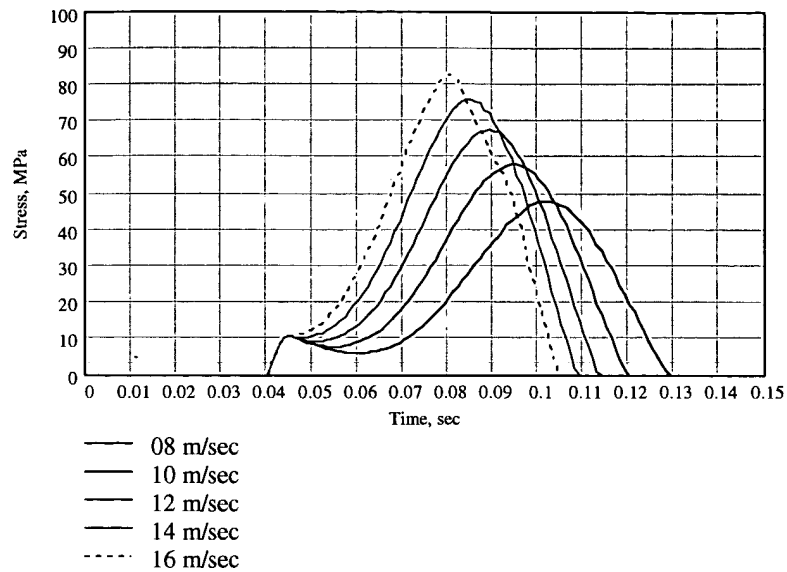


Fig. 8.71. Stresses within the equator zone of the airbag ( $d=0.625$  m;  $k=1.3$ ;  $R=0.4$  m)

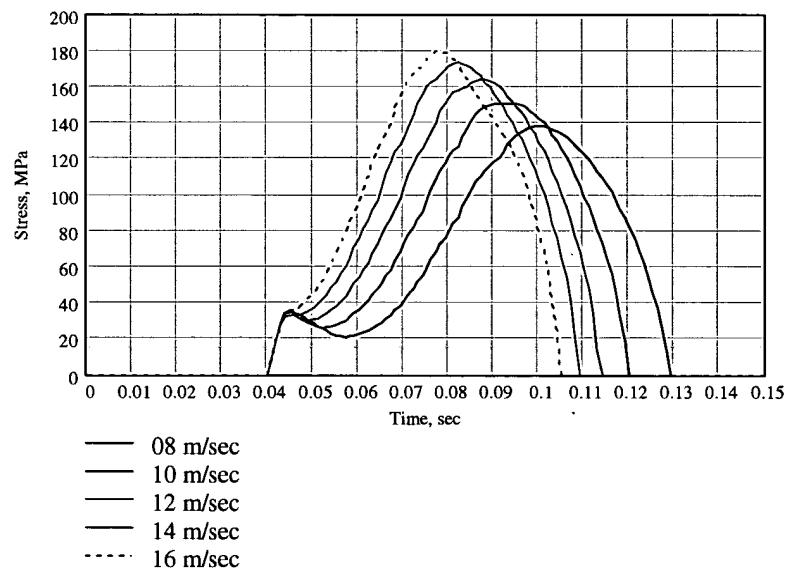


Fig. 8.72. Stresses within the top zone of the airbag ( $d=0.625$  m;  $k=1.3$ ;  $R=0.4$  m)

## 9. CONCLUSIONS

It follows from the results of calculations that the airbag with the diameter of 0.75 m is more efficient than the airbag with the diameter of 0.625 m. The airbag with the diameter of 0.625 m in case of head-on the barrier is completely flattened at the velocity of 16 m/sec, and the occupant strikes against the airbag's bottom.

As expected, the rigid car creates bigger overloads in spite of the soft one.

At the impact velocity of 16 m/sec overloads of about 100 arise for the occupant. This is a mortal level of overloads. The airbag is not an efficient protection of the occupant's life at the above impact velocities.

Stresses within the film are excessively high, so that, the film should be almost 2 times enlarged. At that, stresses will decrease less than twice.

It is very important to note, that reliable results on stresses within the airbag are obtained in the equator zone of the airbag. Stresses within the bottom and top zones of the airbag are determined by irregularity of surfaces, being in contacts with the airbag. The idealized shapes of both the airbag's bottom (flat) and the occupant (paraboloid of revolution) bring to conservative values of stresses. Real stresses should be obtained by experiment.

## **10. REFERENCES**

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